Supervisor Simplification in FMSs: Comparative Studies and New Results Using Petri Nets

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***Abstract*— Modern complex systems require intensive application of sophisticated supervisors. Structural simplification techniques are one of the fundamental researches in the context of flexible manufacturing systems (FMSs). They can reduce implementation cost, mitigate fabrication complexity, and improve reliability. Several typical methods have been developed along this direction. In order to thoroughly explore their effectiveness and performance, we not only conduct a comparison investigation but also develop some new theoretical results. Several analytical results and performance measures are proposed for their qualitative and quantitative comparison. Our approach can assist researchers and practitioners to better comprehend the inherent mechanisms and relative merits of these simplification methodologies as well as their applicability in FMSs. This paper is motivated by FMSs' control; however, it is also applicable to other systems with discrete event controllers.**

***Index Terms*— Automated manufacturing systems, deadlock resolution, inequality analysis, Petri nets, supervisor simplification.**

I. Introduction

DESPITE its long existence, manufacturing still plays essential roles in a nation's economy and security. Thanks to its vitality, many countries show increasing interest in its flexibility and efficiency [7], [9], [25]. To maintain competitiveness, manufacturing needs to accelerate its innovation by investing in various advanced manufacturing infrastructures. Among them, the most promising one is flexible manufacturing systems (FMSs) that bring together many manufacturing technologies including the additive manufacturing techniques, i.e., an industrial version of 3-D printing [13], [14]. They are sophisticated systems combining automation, computation, and communication, supporting adaptive, digital, and high-performance production, providing diverse, high-quality, and customized products, as well as leading forward to a revitalizing manufacturing environment [29]. To accommodate various systems and their variants, FMSs advance manufacturing on flexibility, programmability, and decentralization. Rather than require distinct facilities for separate products, they can rapidly reconfigure themselves for the sake of variation adaption [10], [17]. Their major characterizations are in architectures' evolution and products' diversity [24].

Apart from many benefits, FMSs are cumbersome to be coordinated with supervisors [11], [23]. Their structural complexity prohibits the application of dexterous control mechanisms. As always, a supervisor may involve too many conditional constraints that result in numerous sensors, actuators, and connection nodes [1]–[6], [27], [31]. As a common sense, intricate control system may lead to higher cost but lower reliability [3]. In practice, it is indispensable to simplify these supervisors before their application [13]. Many previous approaches develop various methods for the sake of supervisor simplification; however, a unifying description upon them is still expected [13], [14].

There exist some approaches involving either automata or Petri nets (PNs) to solve this problem. In the paradigm of automata, Su and Wonham [28] proposed a method to reduce the supervisor size and complexity. It provides an economical way to represent the supervisor specification in terms of memory requirement. By taking advantage of some extra enabled events, this method can ensure a supervisor with much fewer states. In the paradigm of PNs, the work in [20] pioneers the supervisor simplification by distinguishing control objects as two categories. The control of one category guarantees the same property of another. This method can produce a supervisor with a much simpler structure. As an extension to [20], Piroddi *et al.* [25] proposed a criterion to identify nonredundant constraints. The method can achieve a supervisor with not only a more compact structure but also more behavior permissiveness. Reference [21] tackles the redundancy issues of a set of inequality constraints. Besides some results to normalize these constraints, it identifies the redundant ones in accordance with structural analysis.

In this paper, our work makes the following key contributions.

1) We make a comparison and contrast investigation upon supervisor simplification in the paradigm of PNs.
2) Our approach further generalizes a well-known P-invariant-based control principle that lays a theoretical

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foundation for supervisor design and deserves further and thoughtful investigation. Supervisor designers can be benefited because our work eases their burden to involve too many control places (monitors, in short). In practice, this implies the reduction in supervisor implementation cost, expedition in system response agility, and enhancement in system performance.

3) Our comparison approach can reveal many inherent rationales about the removal of some redundant constraints during supervisor simplification. Our recent approach shows that inequality analysis can well explain many simplification mechanisms [13]. As a consequence, it is powerful to substantially reduce the supervisors' structure complexity [12]–[16], [18], [19].

This paper is organized as follows. Section II provides a formal but basic description of PNs, whose fundamental definitions and notations are used throughout this paper. Section III is devoted to a special class of PNs. Some knowledge is provided regarding the supervisor simplification. In Section IV, a comparative study is conducted between two typical simplification methods. Moreover, a generalized control principle is developed so as to facilitate further exploration. Section V demonstrates an illustrative example. Section VI concludes this paper.

II. Preliminaries

In this section, we present some notations that are used in the following discussions.

A. Mathematical Basics

Let D be a set. A multiset α over D is defined as a mapping $\alpha : D \to \mathbb{N}$ and can be represented as $\alpha = \cup_{d\in D} \{\alpha(d) \cdot d\}$. We denote $\mathrm{Bag}(D) = \{\alpha |\ \alpha$ is a multiset of $D\}$. In $\mathrm{Bag}(D)$, the following operators are defined. If $\alpha, \alpha' \in \mathrm{Bag}(D)$, then: 1) $\alpha \geq \alpha'$ if $\forall d \in D$ and $\alpha(d) \geq \alpha'(d)$; 2) $\alpha + \alpha' = \sum_{d\in D} (\alpha(d) + \alpha'(d)) \cdot d$; 3) $\max(\alpha, \alpha') = \sum_{d\in D} (\max(\alpha(d), \alpha'(d))) \cdot d$; and 4) $\alpha - \alpha' = \sum_{d\in D} (\alpha(d) - \alpha'(d)) \cdot d$. In the case that $\alpha(d) < \alpha'(d)$ for some $d \in D$, the quantifiers of their differences are negative.

B. Petri Net's Definitions

A PN is $N = (P, T, F, W)$, where P is a set of places, T is a set of transitions, $F \subseteq (P \times T) \cup (T \times P)$ is a set of directed arcs, and $W : (P \times T) \cup (T \times P) \to \mathbb{N} = \{0, 1, 2, \ldots\}$ such that $P \cup T \neq \emptyset$, $P \cap T = \emptyset$, and $W(x, y) = 0$ if $(x, y) \notin F$. If $x, y \in P$ or $x, y \in T$, $W(x, y)$ is undefined. Specially, when $W : F \to \{1\}$, N is said to be ordinary; otherwise, it is general. A marking of N is a mapping $M : P \to \mathbb{N}$. It can be represented by tokens located at various places. We denote the number of tokens in p at M (resp., M_0) by $M(p)$ (resp., $M_0(p)$). Specifically, M_0 denotes the initial marking. A net system with an initial marking is denoted by (N, M_0). Graphically, places, transitions, and tokens are represented by circles, bars, and dots, respectively. As far as a W is nonzero, it is depicted by an arc bridging a pair of place and transition. Its value is labeled by a number, namely, weight, which assigns to each arc a nonnegative integer arc multiplicity; nevertheless, no arc may connect two places or two transitions. By default, the absence of a label for an arc implies that its weight is unity. A PN system's size is defined by $|N| = |P| + |T| + \sum_{p\in P} M_0(p)$.

C. Structural Properties

A PN is said to be pure if $\forall x, y \in P \cup T : W(x, y) \neq 0 \Rightarrow W(y, x) = 0$. The preset of a node $x \in P \cup T$ is defined as ${}^\bullet x = \{y \in P \cup T \mid (y, x) \in F\}$. Its postset $x^\bullet = \{y \in P \cup T \mid (x, y) \in F\}$. A PN N is a state machine if $W : F \to \{1\}$ and $\forall t \in T$, $|{}^\bullet t| = |t^\bullet| = 1$. It is a marked graph if $W : F \to \{1\}$ and $\forall p \in P$, $|{}^\bullet p| = |p^\bullet| = 1$. A PN N's input incidence matrix is $[N^-]_{|P|\times|T|} = [W(p_i, t_j)]$ and the output one is $[N^+]_{|P|\times|T|} = [W(t_j, p_i)]$. Its incidence matrix is $[N]_{|P|\times|T|} = [N^+]_{|P|\times|T|} - [N^-]_{|P|\times|T|}$. $[N_{p_i}]_{|P|\times 1}$ (resp., $[N^-_{p_i}]_{|P|\times 1}$, $[N^+_{p_i}]_{|P|\times 1}$) is the ith row of $[N]_{|P|\times|T|}$ (resp., $[N^-]_{|P|\times|T|}$, $[N^+]_{|P|\times|T|}$). A path is an ordered string $\langle x_1, x_2, \ldots, x_n\rangle$ such that: 1) $\{x_1, x_2, \ldots, x_n\} \subseteq P \cup T$ and 2) $\forall i \in \mathbb{N}_{n-1} = \{1, 2, \ldots, n-1\}$, $x_{i+1} \in x_i^\bullet$. A simple path is an ordered string whose all nodes are different. A circuit is a simple path such that $x_1 = x_n$. Evidently, a circuit rules out any duplicated entries except the first and last entries. A PN is strongly connected if there exists a directed path from every node to every other one in $P \cup T$.

D. Dynamic Properties

A transition t is enabled at M, denoted by $M\ [t\rangle$, if $\forall p \in {}^\bullet t$, $M(p) \geq W(p, t)$. By $M\ [t\rangle\ M'$, we mean that the firing of t at M leads to M'. Given a marking M, t can fire if it is enabled at M. Another marking M' is reachable from M, denoted by $M\ [\sigma\rangle\ M'$, if there exists a firing sequence $\sigma = \langle t_1\ t_2 \ldots t_n\rangle$ such that $M\ [t_1\rangle\ M_1 \ldots [t_n\rangle\ M'$. A vector $\overrightarrow{\sigma}$ is a $|T|$-dimensional firing count vector, where $\overrightarrow{\sigma}(t)$ states the number of t's appearances in σ. Precisely, this evolution can be described by $M' = M + [N] \cdot \overrightarrow{\sigma}$. The set of all markings reachable from M_0 is denoted by $R(N, M_0)$. It follows a necessary reachability condition, i.e., $M = M_0 + [N] \cdot \overrightarrow{\sigma}$. When $|\sigma| = 1$, we have $M\ [t\rangle\ M'$, implying t's firing at M can lead to M'. A net system (N, M_0) is bounded if $\exists k \in \mathbb{N}^+ = \mathbb{N} \setminus \{0\}, \forall M \in R(N, M_0), \forall p \in P$, and $M(p) \leq k$. A transition $t \in T$ is live under M_0 if $\forall M \in R(N, M_0)$, $\exists M' \in R(N, M)$, $M'\ [t\rangle$ holds. A transition t is dead at $M \in R(N, M_0)$ if $\nexists M' \in R(N, M)$ so that $M'\ [t\rangle$ holds. A net system (N, M_0) is deadlock free if $\forall M \in R(N, M_0)$, $\exists t \in T$, $M\ [t\rangle$. It is livelock if it is deadlock free and $\exists t \in T$ so that t is dead at $M \in R(N, M_0)$. A net system (N, M_0) is live if $\forall t \in T$, t is live under M_0.

E. Fundamental Objects

A nonempty set $S \subseteq P$ (resp., $Q \subseteq P$) is a siphon (resp., trap) if ${}^\bullet S \subseteq S^\bullet$ (resp., $Q^\bullet \subseteq {}^\bullet Q$). A strict minimal siphon is a siphon containing neither other siphon nor trap except itself. The sum of token numbers in S is denoted by $M(S)$, where $M(S) = \sum_{p\in S} M(p)$. A subset $S \subseteq P$ is marked by M if $M(S) > 0$. A siphon is undermarked if $\nexists t \in S^\bullet$ can fire.

A P (resp., T) vector is a column vector $I : P$ (resp., $J : T$) $\rightarrow \mathbb{Z}$ indexed by P (resp., T), where $\mathbb{Z}$ is the set of integers. A P-vector $I \neq 0$ becomes a P-invariant if $[N]^T \cdot I = 0$, where 0 means a vector of zeros. By $I \geq 0$, we mean that $\forall p \in P$, $I(p) \geq 0$, and $\exists p \in P$, $I(p) > 0$. A P-invariant is called a P-semiflow if $I \geq 0$. A set $\|I\| = \{p \in P \mid I(p) \neq 0\}$ is called the support of I. A set $\|I\|^+ = \{p \in P \mid I(p) > 0\}$ (resp., $\|I\|^- = \{p \in P \mid I(p) < 0\}$) is called the positive (resp., negative) support of I. A P-semiflow I (resp., T-semiflow J) is said to be minimal if there exists no other P-semiflow I' (resp., T-semiflow J') such that $\|I\| \supset \|I'\|$ (resp., $\|J\| \supset \|J'\|$). For economy of space, $\sum_{p \in P} M(p) \cdot p$ (resp., $\sum_{p \in P} I(p) \cdot p$, $\sum_{t \in T} J(t) \cdot t$) is used to denote vector M (resp., I, J). A net system (N, M_0) is conservative (resp., consistent) if $\exists I > 0$ (resp., $\exists J > 0$) so that $I^T \cdot [N] = 0^T$ (resp., $[N] \cdot J = 0$).

III. PN Modeling and Controlling of FMSs

To facilitate the description and analysis, our approach is demonstrated through a class of structurally special PNs, namely, system of sequential systems with shared resources (S^4R) [26], [29]. In their paradigm, any FMS is an interaction between processes and resources. It contains K process types $\mathbb{J} = \{\mathcal{J}_i\}$ and L resource types $\mathbb{R} = \{\mathcal{R}_j\}$, where $i \in \mathbb{N}_K$ and $j \in \mathbb{N}_L$. Moreover, $\mathcal{J}_i$ defines a series of concurrent and/or sequential job stages, as well as $\mathcal{R}_j$ defines a resource whose capacity $\mathcal{C}_j \in \mathbb{N}^+$. By capacity, e.g., $\mathcal{C}_j$, we mean how many slots a resource, e.g., $\mathcal{R}_j$, holds. A job stage p_{ik} interacts with greater than one but less than L resource types in a conjunctive way, denoted by an L-dimensional vector $a_{p_{ik}}$, where $a_{p_{ik}}[j]$, $j \in \mathbb{N}_L$, indicates resource $\mathcal{R}_j$'s count required to execute such a job stage. Hereby, conjunctive interaction means each process stage requires not only an arbitrary number of units but also an arbitrary number of types of resources for its successful execution.

In PN framework, each process can be represented by a subnet $N \mid (\{p_{0i}\} \cup P_{A_i}, T_i, F_i)$, which is a strongly connected state machine such that: 1) $P_{A_i} \neq \emptyset$ and $p_{0i} \notin P_{A_i}$; 2) its every circuit contains p_{0i}; and 3) each $p \in P_{A_i}$ corresponds to a job stage of $\mathcal{J}_i$, while each p_{0i} corresponds to the initialization and termination of $\mathcal{J}_i$.

A. S^4R Models

Definition 1: An S^4R is a strongly connected, general, and pure PN $N = (P, T, F, W)$ where the following conditions hold.

1) $P = P_0 \cup P_A \cup P_R$ is a partition such that the following conditions hold.
   a) P_0, P_A, and P_R are called idle, operation (or activity), and resource places, respectively.
   b) $P_0 = \cup_{i \in \mathbb{N}_K} \{p_{0i}\}$.
   c) $P_A = \cup_{i \in \mathbb{N}_K} P_{A_i}$, where for each $i \in \mathbb{N}_K$, $P_{A_i} \neq \emptyset$, and for each $i, j \in \mathbb{N}_K$, $i \neq j$, $P_{A_i} \cap P_{A_j} = \emptyset$.
   d) $P_R = \{r_1, r_2, \ldots, r_L\}$.
2) $T = \cup_{i \in \mathbb{N}_K} T_i$, where for each $i \in \mathbb{N}_K$, $T_i \neq \emptyset$, and for each $i, j \in \mathbb{N}_K$, $i \neq j$, $T_i \cap T_j = \emptyset$.
3) For each $i \in \mathbb{N}_K$, subnet $\overline{N_i} = N \mid (\{p_{0i}\} \cup P_{A_i}, T_i, F_i)$ is a strongly connected state machine such that every circuit contains p_{0i}.
4) For each $r \in P_R$, there exists a unique minimal P-semiflow $X_r \in \mathbb{N}^{|P|}$ such that $\{r\} = \|X_r\| \cap P_R$, $P_0 \cap \|X_r\| = \emptyset$, $P_A \cap \|X_r\| \neq \emptyset$, and $X_r(r) = 1$, where $\mathbb{N}^{|P|}$ means $|P|$-dimensional vectors whose each component belongs to $\mathbb{N}$.
5) $P_A = \cup_{r \in P_R} (\|X_r\| \setminus \{r\})$.

Definition 2: In an S^4R (N, M_0), M_0 is an acceptable initial marking in N if: 1) $\forall p_0 \in P_0$, $M_0(p_0) \geq 1$; 2) $\forall p \in P_A$, $M_0(p) = 0$; and 3) $\forall r \in P_R$, $\forall p \in P_A$, $M_0(r) \geq X_r(p)$. Given N with an acceptable initial marking M_0, we say N is acceptably marked.

For $p \in P$, it denotes an arbitrary place. For $p \in P_A$, it denotes a specific place that physically represents an operation stage. To economize symbols, p is employed in this way without confusion. For $p_0 \in P_0$, it denotes an arbitrary idle place. For $p_{0i} \in P_0$, it specifically denotes the idle place with regard to the ith process.

Given an arbitrary marking $M \in R(N, M_0)$, a transition t is process enabled if $M(^\bullet t \cap P_A) > 0$. Definition 1 ensures that each process corresponds to a state machine. In accordance with the definition of state machine, we have $|^\bullet t \cap P_A| = 1$. Correspondingly, t is resource enabled by $\forall r \in {}^\bullet t \cap P_R$ if $M(r) \geq W(r, t)$. In the rest of this paper, (N, M_0) is an acceptably marked S^4R.

Definition 3: In an S^4R (N, M_0), let $r \in P_R$ be a resource place in (N, M_0). The set of holders of r is the support of a minimal P-semiflow X_r without r, i.e., $H(r) = \|X_r\| \setminus \{r\}$. Clearly, $H(r)$ contains only operation places due to $\|X_r\| \cap P_R = \{r\}$.

By X_r, we mean the minimal P-semiflow whose support is r along with all its holders. Each entry of X_r represents either r itself or how many copies of r are requested by a particular place except r itself, i.e., $p \in P \setminus \{r\}$. By $X_r(r)$, we mean one of the X_r's entries, which particularly corresponds to r.

Definition 4: In an S^4R (N, M_0), let S be a siphon. Its complementary set, denoted by $\widetilde{H}(S)$, is the multiset of places with regard to the holders of the resources in S, but do not belong to S.

Let $S_R = S \cap P_R$ and $S_A = S \cap P_A$.

Suppose $H_{S_R} = \cup_{r \in S_R} \|X_r\|$. We have $\widetilde{H}(S) = \cup_{p \in H_{S_R} \setminus S} \{(\sum_{r \in S_R} X_r(p)) \cdot p\}$. The following condition for liveness of S^4R is presented in [29]. (N, M_0) is live if $\nexists M \in R(N, M_0)$ and an undermarked siphon S such that: 1) $\forall r \in S_R$, $M(r) < W(r, t)$; 2) $\forall p \in S_A$, $M(p) = 0$; and 3) $\forall p \in \widetilde{H}(S)$, $M(p) > 0$.

Under the assumptions that $P_0 = \{p_{01}, p_{02}\}$, $P_{A_1} = \{p_{11}, \ldots, p_{13}\}$, $P_{A_2} = \{p_{21}, \ldots, p_{23}\}$, and $P_R = \{r_1, \ldots, r_3\}$, Fig. 1 shows an S^4R representing an FMS consisting of three resource types $\mathcal{R}_1, \ldots, \mathcal{R}_3$ with capacities $\mathcal{C}_1 = \mathcal{C}_2 = \mathcal{C}_3 = 2$, and supporting two job types $\mathcal{J}_1$ and $\mathcal{J}_2$. Job type $\mathcal{J}_1$ (resp., $\mathcal{J}_2$) is defined by a set of ordered job stages $\{p_{01}, p_{11}, \ldots, p_{13}\}$ (resp., $\{p_{02}, p_{21}, \ldots, p_{23}\}$). The conjunctive resource requirements associated with various job stages are as follows: $a_{p_{11}} = [1\ 0\ 0]^T$, $a_{p_{12}} = [0\ 1\ 0]^T$,

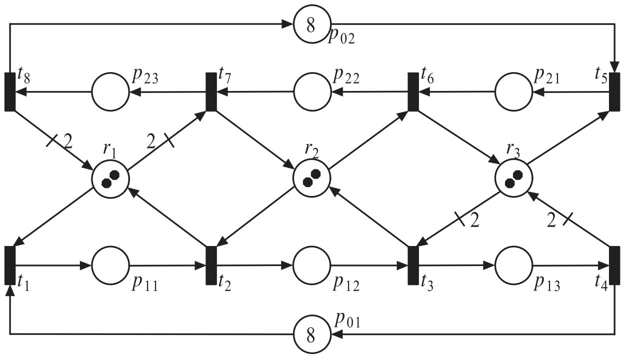

Fig. 1. Example S^4R.

$a_{p_{13}} = [0\ 0\ 2]^T$, $a_{p_{21}} = [0\ 0\ 1]^T$, $a_{p_{22}} = [0\ 1\ 0]^T$, and $a_{p_{23}} = [2\ 0\ 0]^T$. The P-semiflows corresponding to the resources are as follows: $X_{r_1} = [0\ 1\ 0\ 0\ 0\ 0\ 0\ 2\ 1\ 0\ 0]^T$, $X_{r_2} = [0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0]^T$, and $X_{r_3} = [0\ 0\ 0\ 2\ 0\ 1\ 0\ 0\ 0\ 0\ 1]^T$.

B. Supervisory Control

A large body of results have been developed in terms of supervisory control techniques (SCTs) [23], particularly, in the framework of state-based control. They are control mechanisms determining the feasibility to fire a controllable and enabled transition. Their purpose is to ensure that no forbidden state is reached. Ideally, little intervention should be introduced to a system's normal behavior. Among them, a popular one is generalized mutual exclusive constraints (GMECs) [11].

GMECs are initially proposed in [11] as a linear inequality constraint $l^T \cdot M \leq b$, where M means an arbitrary reachable marking, while l and b are the integer vector and scalar, respectively [11]. This constraint can be enforced with a monitor denoted by p_c. A monitor should be superimposed on a net structure according to a row of incidence matrix $[N_{p_c}] = -\, l^T \cdot [N]$. The initial marking of the monitor, denoted by $M_0(p_c)$, is determined by $M_0(p_c) = b - l^T \cdot M_0$. Mathematically, a negative initial marking is unacceptable for a monitor. This means $M_0(p_c) \geq 0$.

In other words, the involvement of p_c is to ensure $l^T \cdot M \leq b$. By adding on the left-hand side a nonnegative scalar, i.e., $M(p_c)$, we have $l^T \cdot M + M(p_c) = b$. In PNs, this equality corresponds to a P-invariant, i.e., $[l^T \mid +1]^T$, where $+1$ corresponds the incidence matrix $[N]$'s additional row, i.e., $[N_{p_c}]$. As a consequence, we have $[l^T \mid +1] \cdot [N^T \mid N_{p_c}^T]^T = 0^T$, which can be expanded as $l^T \cdot [N] + [N_{p_c}] = 0^T$. Obviously, this leads to $[N_{p_c}] = -l^T \cdot [N]$. Since $l^T \cdot M + M(p_c) = b$ is always true, we consider the initial marking M_0 at which $l^T \cdot M_0 + M_0(p_c) = b$. This results in $M_0(p_c) = b - l^T \cdot M_0$.

Proposition 1: Let I be a P-invariant and S be a siphon. S is controlled by I under M_0 if $I^T \cdot M_0 > \Delta_S$, $I(p) \leq 0$ for all $p \in P \setminus S$ hold, and $\Delta_S \in \mathbb{R}^+ \cup \{0\}$, where $\mathbb{R}^+$ denotes the positive real numbers excluding 0.

For details of proof, please refer to [22].

Remark: There are three major points for the P-invariant principle's implementation. First, there must be a P-invariant I such that $I^T \cdot [N] = 0^T$. Second, for $\forall p \in P \backslash S$, $I(p) \leq 0$. Third, we have that $I^T \cdot M_0 > \Delta_S$ holds.

Seeming to be simple, SCTs can be quite computationally intensive because of the huge number of GMECs in practice. A typical scenario is the SCTs to realize system liveness. This necessitates the association and implementation of GMECs to siphons whose number increases exponentially with the system size. Clearly, it is of interest to simplify supervisor when necessary. An important motivation is to condense the supervisor size so that system cost can be reduced whereas system reliability can be increased. In this endeavor, there are primarily two approaches, i.e., elementary siphons [14], [20] and inequality analysis [13]. In the sequel, we abbreviate them as $\mathscr{E}$- and $\mathscr{I}$-methods, respectively. Their basic results are as follows.

C. Simplification via $\mathscr{E}$-Method

Definition 5: In an S^4R (N, M_0), let $S \subseteq P$ be a siphon. ϑ_S is the weighted characteristic P-vector of S if $\forall p \in S$, $\vartheta_S(p) = \sum_{r \in S_R} X_r(p)$; otherwise, $\vartheta_S(p) = 0$. $\gamma_S^T = \vartheta_S^T \cdot [N]$ is its weighted characteristic T-vector.

For any $p = r \in S_R$, we have $\vartheta_S(r) = 1$, which means r is a component in siphon S. For any $p \in S_A$, we have $\vartheta_S(p) = \sum_{r \in S_R} X_r(p)$, which represents how many units of various resources in S_R are required to execute a stage denoted by p. Apparently, for any $p \in S$, we have $\vartheta_S(p) > 0$. For any $p \in P \setminus S$, we have $\vartheta_S(p) = 0$.

In the case $p = r$, $X_r(p) = 1$ (resp., 0) means that r is (resp., is not) a component in $\|X_r\|$. In the case $p \in P_A$, $X_r(p)$ represents how many copies of r are required to execute a stage denoted by p.

Definition 5 shows that each siphon leads to a weighted characteristic P-vector, i.e., $\vartheta_{S|P|\times 1}$, and a weighted characteristic T-vector, i.e., $\gamma_{S|T|\times 1}$, respectively. As a consequence, for a set of siphons Π, there are $|\Pi|$ number of weighted characteristic T-vectors, i.e., $\gamma_{S_1}, \gamma_{S_2}, \ldots, \gamma_{S_{|\Pi|}}$, which constitute a vector space. For $i \in \mathbb{N}_{|\Pi|}$, γ_{S_i}'s jth member, $\gamma_{S_i}(j)$, indicates the connectivity between a monitor, i.e., p_{c_i}, and a transition, i.e., t_j. If $\gamma_{S_i}(j) > 0$, there is an arc with weight $\gamma_{S_i}(j)$ from t_j to p_{c_i}. If $\gamma_{S_i}(j) < 0$, there is an arc with weight $-\gamma_{S_i}(j)$ from p_{c_i} to t_j. If $\gamma_{S_i}(j) = 0$, there is no connectivity between p_{c_i} and t_j.

Definition 6: In an S^4R (N, M_0), given a set of siphons Π, their weighted characteristic T-vectors form a vector space.

1) $S_1, \ldots, S_n$ (resp., $S_1, \ldots, S_{n+m}$) are called elementary siphons of Π if their weighted characteristic T-vectors form a base of Π.
2) For $\forall i$, $j \in \mathbb{N}_{n+m}$, $\alpha_i > 0$ and $\alpha_j > 0$.
   a) $S \in \Pi_1$ is strongly dependent on elementary siphons $S_1, \ldots, S_n$, if $\gamma_S = \sum_{i=1}^{n} \alpha_i \cdot \gamma_{S_i}$.
   b) $S \in \Pi_2$ is weakly dependent on elementary siphons $S_1, \ldots, S_{n+m}$ if $\gamma_S = \sum_{i=1}^{n} \alpha_i \cdot \gamma_{S_i} - \sum_{j=n+1}^{n+m} \alpha_j \cdot \gamma_{S_j}$.

3) If S is dependent on $S_1, \ldots, S_n$ (resp., $S_1, \ldots, S_{n+m}$), we say that $S_1, \ldots, S_n$ (resp., $S_1, \ldots, S_{n+m}$) are its elementary siphons.

If S is controlled, we mean S follows Proposition 1; thus, it cannot be undermarked. If S is strongly (resp., weakly) dependent on $S_1, \ldots, S_n$ (resp., $S_1, \ldots, S_{n+m}$), we say that $S_1, \ldots, S_n$ (resp., $S_1, \ldots, S_{n+m}$) are elementary siphons of S. Given a set of elementary siphons $S_1, \ldots, S_n$ (resp., $S_1, \ldots, S_{n+m}$), we denote their strongly (resp., weakly) dependent siphons by Π_1 and Π_2, respectively.

Theorem 1: In an S^4R (N, M_0), let α_i, ξ_{S_i}, $\Delta_S \in \mathbb{R}^+ \cup \{0\}$.

1) A siphon $S \in \Pi_1$ is controlled if its elementary siphons $S_1, \ldots, S_n$ are controlled by the addition of control places $p_{c_1}, \ldots, p_{c_n}$ with $M_0(S) > \sum_{i=1}^{n} \alpha_i \cdot M_0(S_i) - \sum_{i=1}^{n} \alpha_i \cdot \xi_{S_i} + \Delta_S$.
2) A siphon $S \in \Pi_2$ is controlled if $S_1, \ldots, S_{n+m}$ are controlled by the addition of control places $p_{c_1}, \ldots, p_{c_{n+m}}$ with $M_0(S) > \sum_{i=1}^{n} \alpha_i \cdot M_0(S_i) - \sum_{i=1}^{n} \alpha_i \cdot \xi_{S_i} + \Delta_S$.

For the proof of Theorem 1, please refer to [14].

In Theorem 1, Δ_S is a number to ensure that S cannot be undermarked. For its valuation, one can resort to [7], [29], and [30] for a proper option.

D. Simplification via $\mathscr{I}$-Method

Note that the results herein are some minor repetition of [13]. Their presence is necessary to make this paper self-contained. Except for these results, others relevant to our inequality analysis techniques are brand new results.

Suppose $L = [l_1\ l_2 \ldots l_n]$ and $B = [b_1\ b_2 \ldots b_n]^T$. $L^T \cdot M \leq B$ means n GMECs. Among them, some are dependent on others.

Definition 7: In an S^4R (N, M_0), let $L^T \cdot M \leq B$ be a set of inequalities, $\mathcal{M} = \{M | l_i^T \cdot M \leq b_i, \forall i \in \mathbb{N}_n\}$, and $\mathcal{M}_{\mathbb{N}_n \setminus \{k\}} = \{M | l_i^T \cdot M \leq b_i, \forall i \in \mathbb{N}_n - \{k\}\}$. $l_k^T \cdot M \leq b_k$ is said to be dependent on other inequalities if $\mathcal{M} = \mathcal{M}_{\mathbb{N}_n \setminus \{k\}}$.

Theorem 2: In an S^4R (N, M_0), let $L^T \cdot M \leq B$ be a set of inequalities and $k \in \mathbb{N}_n$. $l_k^T \cdot M \leq b_k$ is dependent on the others if there exist $n-1$ nonnegative coefficients α_i, $i \in \mathbb{N}_n \setminus \{k\}$ such that $l_k \leq \sum_{i \in \mathbb{N}_n \setminus \{k\}} \alpha_i \cdot l_i$ and $b_k \geq \sum_{i \in \mathbb{N}_n \setminus \{k\}} \alpha_i \cdot b_i$.

For the proof of Theorem 2, please refer to [13].

Straightforwardly, Theorem 2 provides a way to identify and remove these dependent inequalities if there is any. In the case that no dependent ones can be identified among all the original inequalities, there are two cases. First, if the physical system does not allow, one can do nothing and all the inequalities are independent. Second, if the physical system allows, one can either increase the right-hand vector or decrease the left-hand scalar of some inequalities. In our circumstance, we are mostly in the second case. This is reasonable thanks to Theorem 2, which claims that $l_k^T \cdot M \leq b_k$ is dependent on the others when there exist $n-1$ nonnegative coefficients α_i, $i \in \mathbb{N}_n \setminus \{k\}$ such that $l_k \leq \sum_{i \in \mathbb{N}_n \setminus \{k\}} \alpha_i \cdot l_i$ and $b_k \geq \sum_{i \in \mathbb{N}_n \setminus \{k\}} \alpha_i \cdot b_i$. As a consequence, from the claim that $l_k^T \cdot M \leq b_k$ is independent, we can deduce either $l_k \nleq \sum_{i \in \mathbb{N}_n \setminus \{k\}} \alpha_i \cdot l_i$ and/or $b_k \ngeq \sum_{i \in \mathbb{N}_n \setminus \{k\}} \alpha_i \cdot b_i$. Thus, in order to make $l_k^T \cdot M \leq b_k$ dependent, we ought to increase one or more l_i, $i \in \mathbb{N}_n \setminus \{k\}$ so that $l_k \leq \sum_{i \in \mathbb{N}_n \setminus \{k\}} \alpha_i \cdot l_i$ and/or decrease one or more b_i, $i \in \mathbb{N}_n \setminus \{k\}$ so that $b_k \geq \sum_{i \in \mathbb{N}_n \setminus \{k\}} \alpha_i \cdot b_i$.

Algorithm 1 Identification of Independent and Dependent Inequalities

1 Input: $L^T \cdot M \leq B$;
2 Output: $L_{\mathcal{I}}^T \cdot M \leq B_{\mathcal{I}}$ and $L_{\mathcal{D}}^T \cdot M \leq B_{\mathcal{D}}$ with $L_{\mathcal{I}} = [l_{\omega_1}\ l_{\omega_2}\ \ldots\ l_{\omega_m}]$, $B_{\mathcal{I}} = [b_{\omega_1}\ b_{\omega_2}\ \ldots\ b_{\omega_m}]^T$, $L_{\mathcal{D}} = [l_{\mu_1}\ l_{\mu_2}\ \ldots\ l_{\mu_{n-m}}]$, and $B_{\mathcal{D}} = [b_{\mu_1}\ b_{\mu_2}\ \ldots\ b_{\mu_{n-m}}]^T$ where $\{\mu_1, \mu_2, \ldots, \mu_{n-m}\} \subseteq \mathbb{N}_n \setminus \{\omega_1, \omega_2, \ldots, \omega_m\}$;
3 $i = 1$, $m = 1$, $\mathcal{L}_{\mathcal{I}} := \emptyset$, $\mathcal{L}_{\mathcal{D}} := \mathcal{L}$, $\mathcal{B}_{\mathcal{I}} := \emptyset$, and $\mathcal{B}_{\mathcal{D}} := \mathcal{B}$;
4 Arrange all the elements in $\mathcal{L}$ according to the descending (resp., ascending) order of $|||l|||$ for the max-$\mathcal{L}_{\mathcal{I}}$ (resp., min-$\mathcal{L}_{\mathcal{I}}$), respectively;
5 **while** $i \leq |\mathcal{L}|$ **do**
6   **if** $\exists \alpha_j \geq 0$ so that $l_i \leq \sum_{j=1}^{m} \alpha_j \cdot l_{\omega_j}$ **then**
7   Goto Step 10;
8   **else**
9     $m := m + 1$, $\mathcal{L}_{\mathcal{I}} := \mathcal{L}_{\mathcal{I}} \cup \{l_i\}$, $\mathcal{L}_{\mathcal{D}} := \mathcal{L}_{\mathcal{D}} \setminus \{l_i\}$, $\mathcal{B}_{\mathcal{I}} := \mathcal{B}_{\mathcal{I}} \cup \{b_i\}$, and $\mathcal{B}_{\mathcal{D}} := \mathcal{B}_{\mathcal{D}} \setminus \{b_i\}$;
10   $i := i + 1$.

Let $\mathcal{L} = \cup_{i \in \mathbb{N}_n} \{l_i\}$. Simplifying $L^T \cdot M \leq B$ is to determine $\Omega = \{\omega_1, \omega_2, \ldots, \omega_m\} \subseteq \mathbb{N}_n$ such that $\mathcal{L}_{\mathcal{I}} = \cup_{i \in \Omega} \{l_i\} \subseteq \mathcal{L}$ and $\mathcal{L}_{\mathcal{D}} = \mathcal{L} \setminus \mathcal{L}_{\mathcal{I}}$ where: 1) $\forall \omega_k \in \Omega$, $\nexists \alpha_i \geq 0$ such that $l_{\omega_k} = \sum_{i \in \Omega \setminus \{\omega_k\}} \alpha_{\omega_i} \cdot l_{\omega_i}$; and 2) $\forall \mu_k \in \mathbb{N}_n \setminus \Omega$, $\exists \alpha_i \geq 0$, $l_{\mu_k} \leq \sum_{i \in \mathbb{N}_m} \alpha_i \cdot l_{\omega_i}$. Apparently, $\mathcal{L}_{\mathcal{I}}$ is not unique, leading to its two options, i.e., max-$\mathcal{L}_{\mathcal{I}}$ and min-$\mathcal{L}_{\mathcal{I}}$, respectively.

Let $i \in \Omega$, $j = \mathbb{N}_n \setminus \Omega$. $\{l_{\omega_1}, l_{\omega_2}, \ldots, l_{\omega_m}\}$ is called a max-$\mathcal{L}_{\mathcal{I}}$ (resp., min-$\mathcal{L}_{\mathcal{I}}$) if $\forall i$, j, $|||l_{\omega_i}||| \geq |||l_{\omega_j}|||$ (resp., $|||l_{\omega_i}||| \leq |||l_{\omega_j}|||$).

Similar to $\mathcal{L}$, we have $\mathcal{B} = \cup_{i \in \mathbb{N}_n} \{b_i\}$, $\mathcal{B}_{\mathcal{I}} = \cup_{i \in \Omega} \{b_i\}$, and $\mathcal{B}_{\mathcal{D}} = \cup_{\mathbb{N}_n \setminus \Omega} \{b_i\}$. L_D, B_D, and B_I denote vectors, while $\mathcal{L}_D$, $\mathcal{B}_D$, and $\mathcal{B}_I$ denote sets.

With these notations, our simplification method can be formalized as the following two algorithms [13]. Specifically, Algorithm 1 distinguishes all inequalities by independent and dependent ones. Algorithm 2 retrieves these independent ones by adjusting their right-hand scalars.

Based on Algorithm 1, the inequality set is divided into two disjoint sets, i.e., $L_{\mathcal{I}}$ and $L_{\mathcal{D}}$. The computational complexity is polynomial with regard to the number of inequalities. Using Theorem 2, we can decrease the right-hand scalars such that the ones in $L_{\mathcal{D}}$ can be ignored during the supervisor synthesis process. Algorithm 1 distinguishes the independent and dependent inequalities without resorting to integer programming techniques whose computational complexity proves to be exponential. Its execution involves only a limited number of comparisons between some coefficient vectors and scalars.

In accordance with the fundamental principles provided by theoretical results, Algorithms 1 and 2 formalize two optional solutions to tackle supervisor simplification problems. They are max-$\mathcal{L}_{\mathcal{I}}$ and min-$\mathcal{L}_{\mathcal{I}}$, respectively. The former leads to

Algorithm 2 Supervisor Simplification

```
Input: L^T · M ≤ B;
Output: L_I · M ≤ B_I;
while New dependent Inequalities can be Found and
  Removed by Algorithm 1 do
    Identify L_I and L_D using Algorithm 1;
    i = 0;
    while i < n − m do
        Find α_{ω_j} ≥ 0, j ∈ N_m, such that l_{μ_i} ≤ Σ_{j=1}^{m} α_{ω_j} · l_{ω_j};
        Decrease b_{ω_j}, j ∈ N_m, such that b_{μ_i} ≥ Σ_{j=1}^{m} α_{ω_j} · b_{ω_j}.
```

a concise, if not the simplest, supervisor without considering the permissiveness matter, whereas the latter leads to more, if not the most, permissive supervisor with an as compact as possible structure. In particular, for the latter, one can arbitrarily remove the inequalities whose removal has no influence upon the system behavior. For clarity, we are only concerned by max-$\mathcal{L}_{\mathcal{I}}$ throughout this paper.

In the sequel, we make a theoretical comparative study between $\mathscr{E}$- and $\mathscr{I}$-methods. This approach shows that $\mathscr{I}$-method subsumes $\mathscr{E}$-method in terms of their explanation and control capabilities. Therefore, $\mathscr{I}$-method is more general and advantageous than $\mathscr{E}$-method. This means that ours is superior to the one proposed in [14]. Specifically, $\mathscr{E}$-method explains and performs well in the case that there are only strongly dependent siphons besides elementary ones. Nevertheless, it fails in the case that there are some weakly dependent siphons. Since it is rare that there are a set of elementary siphons whose dependent counterparts are all strongly dependent ones. $\mathscr{E}$-method's applicability is doubtful in reality. As a contrast, $\mathscr{I}$-method covers both the strong and weak dependence cases, leading to a very robust, reliable, and consistent control technique in various PN models.

IV. Comparative Study on $\mathscr{E}$- and $\mathscr{I}$-Methods

According to the above description, both inequality analysis and elementary siphon techniques are derived from GMECs. In the siphon-based control domain, it is $l'^T \cdot M \geq b'$, which can be converted to $l^T \cdot M \leq b$ [32]. A comparative study upon them can assess most results with regard to supervisor simplification. More importantly, we provide a general framework under which all simplification methodologies are unified. Apart from providing a more unifying framework, this approach enables us to develop the theoretical results in the semantics of supervisor simplification.

A. Mathematical Analysis

The above analysis indicates the necessity to compare the simplification approaches in the paradigm of inequality analysis and elementary siphons. To further clarify this point, we present two theorems.

Theorem 3: In an S^4R (N, M_0), let $S_1, \ldots, S_n$ and S be a set of elementary and a strongly dependent siphon(s), respectively. $l_1^T \cdot M \leq b_1$, $l_2^T \cdot M \leq b_2, \ldots, l_n^T \cdot M \leq b_n$, and $l^T \cdot M \leq b$ are their GMECs. Then, $l = \sum_{i=1}^{n} \alpha_i \cdot l_i$, where $\alpha_i \in \mathbb{R}^+ \cup \{0\}$.

Proof: According to Definition 6, we have $\gamma_S = \sum_{i=1}^{n} \alpha_i \cdot \gamma_{S_i}$. This implies $-l^T \cdot [N] = -\sum_{i=1}^{n} \alpha_i \cdot l_i^T \cdot [N]$. From GMECs, we know that $-l^T \cdot [N]$ corresponds to a monitor, i.e., p_c, implying inequality $l^T \cdot M \leq b$. Therefore, $\gamma_S = \sum_{i=1}^{n} \alpha_i \cdot \gamma_{S_i}$ implies $l = \sum_{i=1}^{n} \alpha_i \cdot l_i$. ■

Theorem 4: In an S^4R (N, M_0), let $S_1, \ldots, S_n$, $S_{n+1}, \ldots, S_{n+m}$ and S be a set of elementary and a weakly dependent siphon(s), respectively. $l_1^T \cdot M \leq b_1$, $l_2^T \cdot M \leq b_2, \ldots, l_n^T \cdot M \leq b_n, \ldots, l_{n+1}^T \cdot M \leq b_{n+1}, \ldots, l_{n+m}^T \cdot M \leq b_{n+m}$, and $l^T \cdot M \leq b$ are their GMECs. Then, $l = \sum_{i=1}^{n} \alpha_i \cdot l_i - \sum_{j=n+1}^{n+m} \alpha_j \cdot l_j$.

Proof: According to Definition 6, we have $\gamma_S = \sum_{i=1}^{n} \alpha_i \cdot \gamma_{S_i} - \sum_{j=n+1}^{n+m} \alpha_j \cdot \gamma_{S_j}$. This implies $-l^T \cdot [N] = -\sum_{i=1}^{n} \alpha_i \cdot l_i^T \cdot [N] + \sum_{j=n+1}^{n+m} \alpha_j \cdot l_j$. Again, $-l^T \cdot [N]$ corresponds to a monitor, i.e., p_c, with $l^T \cdot M \leq b$. Therefore, $\gamma_S = \sum_{i=1}^{n} \alpha_i \cdot \gamma_{S_i} - \sum_{j=n+1}^{n+m} \alpha_j \cdot \gamma_{S_j}$ implies $l = \sum_{i=1}^{n} \alpha_i \cdot l_i - \sum_{j=n+1}^{n+m} \alpha_j \cdot l_j$. ■

In Theorems 3 and 4, if $S_1, \ldots, S_n$ (resp., $S_1, \ldots, S_{n+m}$) are elementary siphons of S, we say S is strongly (resp., weakly) dependent on $S_1, \ldots, S_n$ (resp., $S_1, \ldots, S_{n+m}$). By GMEC of a siphon, we mean that an inequality $l^T \cdot M \leq b$, which is used to prevent such a siphon from being undermarked. Each α_i is a coefficient whose value is a nonnegative real number.

In the general case, we know that $l^T \cdot [N] = \sum_{i=1}^{n} \alpha_i \cdot l_i^T \cdot [N]$ (resp., $l^T \cdot [N] = \sum_{i=1}^{n} \alpha_i \cdot l_i^T \cdot [N] - \sum_{j=n+1}^{n+m} \alpha_j \cdot l_j^T \cdot [N]$) does not necessarily mean $l^T = \sum_{i=1}^{n} \alpha_i \cdot l_i^T$ (resp., $l^T \cdot [N] = \sum_{i=1}^{n} \alpha_i \cdot l_i^T - \sum_{j=n+1}^{n+m} \alpha_j \cdot l_j^T$). Both $l^T = \sum_{i=1}^{n} \alpha_i \cdot l_i^T$ and $\sum_{i=1}^{n} \alpha_i \cdot l_i^T - \sum_{j=n+1}^{n+m} \alpha_j \cdot l_j^T$ mean that $l^T \leq \sum_{i=1}^{n} \alpha_i \cdot l_i^T$ considering that $\sum_{j=n+1}^{n+m} \alpha_j \cdot l_j^T \geq 0$. Moreover, the values of $b_1, b_2, \ldots, b_n$ can be intentionally changed to $b_1', b_2', \ldots, b_n'$ such that $b \geq \sum_{i=1}^{n} \alpha_i \cdot b_i'$. This fulfills the requirements in Theorem 2. As a result, for any set of elementary siphons, their GMECs can be the independent inequalities. On the other hand, their dependent siphons' GMECs become the dependent inequalities.

Proposition 2: In an S^4R (N, M_0), there exist $n + 1$ (resp., $n + m + 1$) siphons' GMECs, i.e., $l^T \cdot M \leq b$, $l_1^T \cdot M \leq b_1$, $l_2^T \cdot M \leq b_2, \ldots, l_n^T \cdot M \leq b_n$ (resp., $l^T \cdot M \leq b$, $l_1^T \cdot M \leq b_1$, $l_2^T \cdot M \leq b_2, \ldots, l_n^T \cdot M \leq b_n$, $l_{n+1}^T \cdot M \leq b_{n+1}, \ldots, l_{n+m}^T \cdot M \leq b_{n+m}$, which do not mean $\forall i \in \mathbb{N}_n$, l_i's corresponding siphons are elementary ones, as well as l's siphon is their dependent one, but satisfy $l \leq \sum_{i=1}^{n} \alpha_i \cdot l_i$.

Proof: According to Theorems 3 and 4, when $\forall i \in \mathbb{N}_n$ (resp., $\forall i \in \mathbb{N}_{n+m}$), l_i's corresponding siphons are elementary ones, whereas l's corresponding siphon is the dependent one, we have $l = \sum_{i=1}^{n} \alpha_i \cdot l_i$ (resp., $l = \sum_{i=1}^{n} \alpha_i \cdot l_i - \sum_{j=n+1}^{n+m} \alpha_j \cdot l_j$). Apparently, this equation is only a coincidence of the equality, i.e., $l \leq \sum_{i=1}^{n} \alpha_i \cdot l_i$. In other words, there exist siphons' GMECs, which do not mean l_i's corresponding siphons are elementary ones as well as l's corresponding siphon is their dependent one such that $l = \sum_{i=1}^{n}\alpha_i \cdot l_i$ (resp., $l = \sum_{i=1}^{n}\alpha_i \cdot l_i - \sum_{j=n+1}^{n+m}\alpha_j \cdot l_j$), but satisfy $l \leq \sum_{i=1}^{n}\alpha_i \cdot l_i$. ■

Theorems 3 and 4 imply that elementary-siphon-based simplification strategy can also be explained in the paradigm of inequality analysis; however, Proposition 2 implies that an inequality-based simplification does not necessarily mean an elementary-siphon-based one. As a result, elementary-siphon-based simplification degrades to a special case of inequality analysis. In other words, inequality analysis is a more general simplification strategy.

Take the PN in Fig. 1 as an instance. There are three siphons, i.e., $S_1 = \{p_{12}, p_{23}, r_1, r_2\}$, $S_2 = \{p_{13}, p_{22}, r_2, r_3\}$, and $S_3 = \{p_{13}, p_{23}, r_1, r_2, r_3\}$. In terms of weighted T-characteristic vector, we have $\gamma_{S_1} = [-1\ 1\ 0\ 0\ 0\ -1\ 1\ 0]^T$, $\gamma_{S_2} = [0\ -1\ 1\ 0\ -1\ 1\ 0\ 0]^T$, and $\gamma_{S_3} = [-1\ 0\ 1\ 0\ -1\ 0\ 1\ 0]^T$. Apparently, we have $\gamma_{S_3} = \gamma_{S_1} + \gamma_{S_2}$, which means that S_1 and S_2 are elementary siphons, whereas S_3 is their strongly dependent one. As well, $\gamma_{S_1} = \gamma_{S_3} - \gamma_{S_2}$ (resp., $\gamma_{S_2} = \gamma_{S_3} - \gamma_{S_1}$) means that S_2 and S_3 (resp., S_1 and S_3) are elementary siphons, whereas S_1 (resp., S_2) is their weakly dependent one. To control $S_1, \ldots, S_3$, we have three inequalities, i.e., $l_1^T \cdot M \leq b_1 \Leftrightarrow M(p_2) + M(p_7) \leq 2$, $l_2^T \cdot M \leq b_2 \Leftrightarrow M(p_3) + M(p_6) \leq 2$, and $l_3^T \cdot M \leq b_3 \Leftrightarrow M(p_2) + M(p_3) + M(p_6) + M(p_7) \leq 4$. According to the inequality analysis, we can select either $l_1^T \cdot M \leq b_1 \bigwedge l_2^T \cdot M \leq b_2$ or $l_3^T \cdot M \leq b_3$ as the independent inequalities. If we select $l_1^T \cdot M \leq b_1 \bigwedge l_2^T \cdot M \leq b_2$ as the independent ones, $l_3^T \cdot M \leq b_3$ will be considered as the dependent one and will be abandoned by Algorithm 1. If we select $l_3^T \cdot M \leq b_3$ as the independent one, $l_3^T \cdot M \leq b_3 = 4$ will be converted to $l_3 \cdot M \leq b_3' = 2$ by Algorithm 2. As a consequence, $l_1^T \cdot M \leq b_1$ and $l_2^T \cdot M \leq b_2$ are considered as the dependent ones and will be abandoned.

B. Comparison and Discussion

In our approach, GMECs are divided into two categories, i.e., the independent and dependent ones. An algorithm is established to retrieve a set of independent inequalities so that the remaining ones can depend on. This provides a succinct and compact way to characterize a simplified supervisor. Unlike inequality analysis, elementary siphons are defined on the basis of P-invariant control principle that is as follows. Note that in Proposition 3 (resp., Proposition 4), M denotes a marking of the plant PN, i.e., $M = R(N, M_0)$. $M^* = [M \mid M(p_{c_1}) \mid M(p_{c_2}) \mid \ldots \mid M(p_{c_n})]$ (resp., $M^* = [M \mid M(p_{c_1}) \mid M(p_{c_2}) \mid \ldots \mid M(p_{c_{n+m}})]$) denotes the marking of PN after adding monitors, i.e., $p_{c_1}, p_{c_2}, \ldots, p_{c_n}$ (resp., $p_{c_1}, p_{c_2}, \ldots, p_{c_{n+m}}$). Apparently, $M_0^* = [M_0 \mid M(p_{c_1}) \mid M(p_{c_2}) \mid \ldots \mid M(p_{c_n})]$ (resp., $M_0^* = [M_0 \mid M(p_{c_1}) \mid M(p_{c_2}) \mid \ldots \mid M(p_{c_{n+m}})]$).

Proposition 3: In an S^4R (N, M_0), let $S_1, \ldots, S_n$ and S be a set of elementary and a strongly dependent siphon(s), respectively. If $S_1, \ldots, S_n$ are controlled by $p_{c_1}, \ldots, p_{c_n}$ and $M_0(S) > \sum_{i=1}^{n}\alpha_i \cdot M_0(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i} + \Delta_S$, S is controlled because of the conformance to P-invariant control principle.

Proof: First, we have P-vector $I = [\vartheta_S^T\ -\alpha_1\ -\alpha_2 \cdots -\alpha_n]^T$. I is a P-invariant because $I^T \cdot [N] = [\vartheta_S^T\ -\alpha_1\ -\alpha_2 \cdots -\alpha_n]^T \cdot [N^T \mid \gamma_{S_1} \mid \gamma_{S_2} \mid \ldots \mid \gamma_{S_n}]^T = \vartheta_S^T \cdot [N] - \alpha_1 \cdot \gamma_{S_1}^T - \alpha_2 \cdot \gamma_{S_2}^T \ldots \alpha_n \cdot \gamma_{S_n}^T = \gamma_S^T - \sum_{i=1}^{n}\alpha_i \cdot \gamma_{S_i}^T = 0^T$.

Second, for $\forall p \in S$, $I(p) = \vartheta_S(p) > 0$, whereas for $\forall p \in P \backslash S$, $I(p) = -\alpha_i \leq 0$.

Third, $I^T \cdot M^* = [\vartheta_S^T\ -\alpha_1\ -\alpha_2 \cdots -\alpha_n]^T \cdot [M \mid M(p_{c_1}) \mid M(p_{c_2}) \mid \ldots \mid M(p_{c_n})] = \vartheta_S^T \cdot M - \alpha_1 \cdot M(p_{c_1}) - \alpha_2 \cdot M(p_{c_2}) - \cdots - \alpha_n \cdot M(p_{c_n}) = \vartheta_S^T \cdot M - \alpha_1 \cdot (M(S_1) - \xi_{S_1}) - \alpha_2 \cdot (M(S_2) - \xi_{S_2}) - \cdots - \alpha_n \cdot (M(S_n) - \xi_{S_n}) = \vartheta_S^T \cdot M - (\sum_{i=1}^{n}\alpha_i \cdot M(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i}) = \vartheta_S^T \cdot M_0 - (\sum_{i=1}^{n}\alpha_i \cdot M_0(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i}) > \Delta_S$. Thus, $\vartheta_S^T \cdot M - (\sum_{i=1}^{n}\alpha_i \cdot M(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i}) > \Delta_S \Rightarrow \vartheta_S^T \cdot M > (\sum_{i=1}^{n}\alpha_i \cdot M(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i}) + \Delta_S \Rightarrow \vartheta_S^T \cdot M > \Delta_S$. The last implication holds because it is definite that $\sum_{i=1}^{n}\alpha_i \cdot M(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i} > 0$.

Clearly, these three parts strictly and respectively follow the three fundamental points in P-invariant control principle in Proposition 1. ■

Remark: From the inequality analysis, a strong dependence case corresponds to $l = \sum_{i=1}^{n} l_i$. In terms of multiset, $\|l\|^* = \cup_{i=1}^{n}\|l_i\|^*$. $\|l\|^*$ denotes $\|l\|$'s multiset form, i.e., $\|l\|^* = \cup_{p \in \|l\|}\ \{l(p) \cdot p\}$. So does $\|l_i\|^*$. In terms of siphon, $\widetilde{H}(S) = \cup_{i \in \mathbb{N}_n}\widetilde{H}(S_i)$. We can decrease b_i to b_i' such that $b = \sum_{i=1}^{n} b_i'$. As a consequence, the control of these elementary siphons can necessarily and sufficiently implement the control of this strongly dependent siphon by default.

Proposition 4: In an S^4R (N, M_0), let $S_1, \ldots, S_n, S_{n+1}, \ldots, S_{n+m}$ and S be a set of elementary and a weakly dependent siphon(s), respectively. There exist some S such that if $S_1, \ldots, S_{n+m}$ are controlled by $p_{c_1}, \ldots, p_{c_{n+m}}$ and $M_0(S) > (\sum_{i=1}^{n}\alpha_i \cdot M_0(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i}) - (\sum_{j=n+1}^{n+m}\alpha_j \cdot M_0(S_j) - \sum_{j=n+1}^{n+m}\alpha_j \cdot \xi_{S_j}) + \Delta_S$ holds, but S is not controlled.

Proof: First, we have P-vector $I = [\vartheta_S^T - \alpha_1 - \alpha_2 \cdots - \alpha_n + \alpha_{n+1} + \alpha_{n+2} \cdots + \alpha_{n+m}]^T$. I is a P-invariant because $I^T \cdot [N] = [\vartheta_S^T - \alpha_1 - \alpha_2 \cdots - \alpha_n + \alpha_{n+1} + \alpha_{n+2} \cdots + \alpha_{n+m}]^T \cdot [N^T \mid \gamma_{S_1} \mid \gamma_{S_2} \mid \ldots \mid \gamma_{S_n} \mid \gamma_{S_{n+1}} \mid \gamma_{S_{n+2}} \mid \ldots \mid \gamma_{S_{n+m}}]^T = \vartheta_S^T \cdot [N] - \alpha_1 \cdot \gamma_{S_1}^T - \alpha_2 \cdot \gamma_{S_2}^T \ldots \alpha_n \cdot \gamma_{S_n}^T + \alpha_{n+1} \cdot \gamma_{S_{n+1}}^T + \alpha_{n+2} \cdot \gamma_{S_{n+2}}^T \ldots \alpha_{n+m} \cdot \gamma_{S_{n+m}}^T = \gamma_S^T - \sum_{i=1}^{n}\alpha_i \cdot \gamma_{S_i}^T + \sum_{j=n+1}^{n+m}\alpha_j \cdot \gamma_{S_j}^T = 0^T$.

Second, for $\forall p \in S$, $I(p) > 0$ whereas $\exists p \in P \backslash S$, $I(p) > 0$ since $\forall p \in \cup_{i \in \mathbb{N}_m}\{p_{c_{n+i}}\}$, we have $I(p) = +\alpha_{n+i} > 0$, where $i \in \mathbb{N}_m$.

Third, $I^T \cdot M^* = [\vartheta_S^T\ -\alpha_1\ -\alpha_2 \cdots -\alpha_n + \alpha_{n+1} + \alpha_{n+2} \cdots + \alpha_{n+m}]^T \cdot [M \mid M(p_{c_1}) \mid M(p_{c_2}) \mid \ldots \mid M(p_{c_n}) \mid M(p_{c_{n+1}}) \mid M(p_{c_{n+2}}) \mid \ldots \mid M(p_{c_{n+m}})] = \vartheta_S^T \cdot M - \alpha_1 \cdot M(p_{c_1}) - \alpha_2 \cdot M(p_{c_2}) - \cdots - \alpha_n \cdot M(p_{c_n}) + \alpha_{n+1} \cdot M(p_{c_{n+1}}) + \alpha_{n+2} \cdot M(p_{c_{n+2}}) + \cdots + \alpha_{n+m} \cdot M(p_{c_{n+m}}) = \vartheta_S^T \cdot M - \alpha_1 \cdot (M(S_1) - \xi_{S_1}) - \alpha_2 \cdot (M(S_2) - \xi_{S_2}) - \cdots - \alpha_n \cdot (M(S_n) - \xi_{S_n}) + \alpha_{n+1} \cdot (M(S_{n+1}) - \xi_{S_{n+1}}) + \alpha_{n+2} \cdot (M(S_{n+2}) - \xi_{S_{n+2}}) + \cdots + \alpha_{n+m} \cdot (M(S_{n+m}) - \xi_{S_{n+m}}) = \vartheta_S^T \cdot M - (\sum_{i=1}^{n}\alpha_i \cdot M(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i}) + (\sum_{j=n+1}^{n+m}\alpha_j \cdot M(S_j) - \sum_{j=n+1}^{n+m}\alpha_j \cdot \xi_{S_j}) = \vartheta_S^T \cdot M_0 -$

$(\sum_{i=1}^{n}\alpha_i \cdot M_0(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i}) + (\sum_{j=n+1}^{n+m}\alpha_j \cdot M_0(S_j) - \sum_{j=n+1}^{n+m}\alpha_j \cdot \xi_{S_j}) > \Delta_S$. Thus, $\vartheta_S^T \cdot M - (\sum_{i=1}^{n}\alpha_i \cdot M(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i}) + (\sum_{j=n+1}^{n+m}\alpha_j \cdot M(S_j) - \sum_{j=n+1}^{n+m}\alpha_j \cdot \xi_{S_j}) > \Delta_S \Rightarrow \vartheta_S^T \cdot M > (\sum_{i=1}^{n}\alpha_i \cdot M(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i}) - (\sum_{j=n+1}^{n+m}\alpha_j \cdot M(S_j) - \sum_{j=n+1}^{n+m}\alpha_j \cdot \xi_{S_j}) + \Delta_S \not\Rightarrow \vartheta_S^T \cdot M > \Delta_S$. The last implication does not necessarily hold because one cannot determine the value of $(\sum_{i=1}^{n}\alpha_i \cdot M(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i}) - (\sum_{j=n+1}^{n+m}\alpha_j \cdot M(S_j) - \sum_{j=n+1}^{n+m}\alpha_j \cdot \xi_{S_j})$. In the case that it is nonnegative, $\vartheta_S^T \cdot M > \Delta_S$ is true; otherwise, it is false.

Thus, S is not controlled because of the nonconformance to P-invariant control principle.

Except the first part, the remaining two parts do not follow the P-invariant control principle because $\exists p \in P\backslash S$ such that $I(p) > 0$ and $I^T \cdot M^* = \vartheta_S^T \cdot M + (\sum_{i=1}^{n}\alpha_i \cdot M(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i}) - (\sum_{j=n+1}^{n+m}\alpha_j \cdot M(S_j) - \sum_{j=n+1}^{n+m}\alpha_j \cdot \xi_{S_j}) > 0$ does not necessarily imply $\vartheta_S^T \cdot M > \Delta_S$. ■

In plain words, Propositions 3 and 4 claim two facts, respectively. First, suppose $S_1, \ldots, S_n$ are elementary siphons of a strongly dependent siphon S. If $S_1, \ldots, S_n$ are controlled, the condition $M_0(S) > \sum_{i=1}^{n}\alpha_i \cdot M_0(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i} + \Delta_S$ can ensure that S is also controlled. Second, suppose $S_1, \ldots, S_{n+m}$ are elementary siphons of a weakly dependent siphon S. If $S_1, \ldots, S_{n+m}$ are controlled, the condition $M_0(S) > (\sum_{i=1}^{n}\alpha_i \cdot M_0(S_i) - \sum_{i=1}^{n}\alpha_i \cdot \xi_{S_i}) - (\sum_{j=n+1}^{n+m}\alpha_j \cdot M_0(S_j) - \sum_{j=n+1}^{n+m}\alpha_j \cdot \xi_{S_j}) + \Delta_S$ cannot ensure that S is controlled.

Remark: From the inequality analysis, a weak dependence case corresponds to $l = \sum_{i=1}^{n}l_i - \sum_{j=n+1}^{n+m}l_j$. In terms of multiset, $\|l\|^* = \cup_{i=1}^{n}\|l_i\|^* - \cup_{j=n+1}^{n+m}\|l_j\|^*$. $\|l\|^*$ denotes $\|l\|$'s multiset form, i.e., $\|l\|^* = \cup_{p\in\|l\|} \{l(p) \cdot p\}$. So do $\|l_i\|^*$ and $\|l_j\|^*$. In terms of siphon, $\widetilde{H}(S) = \cup_{i\in\mathbb{N}_n}\widetilde{H}(S) - \cup_{i\in\mathbb{N}_{n+m}\backslash\mathbb{N}_n}\widetilde{H}(S_i)$. An interesting issue is that this phenomenon is hard to be explored with P-invariant control principle because $\exists p \in P\backslash S$, $I(p) > 0$, which does not match the requirement of P-invariant control principle. Nevertheless, our inequality analysis can well explain such a phenomenon. This is because that the case $l = \sum_{i=1}^{n}l_i - \sum_{j=n+1}^{n+m}l_j$ actually implies that $l \le \sum_{i=1}^{n}l_i$, which still falls in our inequality analysis' description capability. We decrease b_i to b_i', $\forall i \in \mathbb{N}_n$, such that $b = \sum_{i=1}^{n}b_i'$. As a consequence, the control of these elementary siphons can necessarily and sufficiently implement the control of this weakly dependent siphon by default.

Based on the above discussion, we can establish a generalized P-invariant control principle.

Proposition 5: Let I and S be a P-invariant and a siphon, respectively. S is controlled by I under M_0 if: 1) $\forall p \in S$, $I(p) > 0$; 2) $\sum_{p\in S}I(p) \cdot M_0(p) - \sum_{p\in\|I\|^-\cap\{P\backslash S\}} |I(p)| \cdot M_0(p) > \Delta_S$; and 3) $\sum_{p\in\|I\|^+\cap\{P\backslash S\}}(|I(p)| \cdot M_0(p) - |I(p)| \cdot M(p)) > 0$.

Proof: Because I is a P-invariant, we have $I^T \cdot M = \sum_{p\in S}I(p) \cdot M(p) - \sum_{p\in\|I\|^-\cap\{P\backslash S\}} |I(p)| \cdot M(p) + \sum_{p\in\|I\|^+\cap\{P\backslash S\}} |I(p)| \cdot M(p) = I^T \cdot M_0 = \sum_{p\in S}I(p) \cdot M_0(p) - \sum_{p\in\|I\|^-\cap\{P\backslash S\}} |I(p)| \cdot M_0(p) + \sum_{p\in\|I\|^+\cap\{P\backslash S\}} |I(p)| \cdot M_0(p)$. According to the hypothesis that $\sum_{p\in S}I(p) \cdot M_0(p) - \sum_{p\in\|I\|^-\cap\{P\backslash S\}} |I(p)| \cdot M_0(p) > \Delta_S$, we have $\sum_{p\in S}I(p) \cdot M(p) - \sum_{p\in\|I\|^-\cap\{P\backslash S\}} |I(p)| \cdot M(p) + \sum_{p\in\|I\|^+\cap\{P\backslash S\}} |I(p)| \cdot M(p) > \sum_{p\in\|I\|^+\cap\{P\backslash S\}} |I(p)| \cdot M_0(p) + \Delta_S$. This further implies that $\sum_{p\in S}I(p) \cdot M(p) > \sum_{p\in\|I\|^-\cap\{P\backslash S\}} |I(p)| \cdot M(p) + (\sum_{p\in\|I\|^+\cap\{P\backslash S\}}|I(p)| \cdot M_0(p) - \sum_{p\in\|I\|^+\cap\{P\backslash S\}}|I(p)| \cdot M(p)) + \Delta_S = \sum_{p\in\|I\|^-\cap\{P\backslash S\}} |I(p)| \cdot M(p) + \sum_{p\in\|I\|^+\cap\{P\backslash S\}}(|I(p)| \cdot M_0(p) - |I(p)| \cdot M(p)) + \Delta_S$. Considering the hypothesis that $\sum_{p\in\|I\|^+\cap\{P\backslash S\}}(|I(p)| \cdot M_0(p) - |I(p)| \cdot M(p)) > 0$, we have that $\sum_{p\in S}I(p) \cdot M(p) > \Delta_S$. ■

For all $M \in R(N, M_0)$, the third condition, i.e., $\sum_{p\in\|I\|^+\cap\{P\backslash S\}}(|I(p)| \cdot M_0(p) - |I(p)| \cdot M(p)) > 0$, holds. This requires $\forall M \in R(N, M_0)$, $\sum_{p\in\|I\|^+\cap\{P\backslash S\}}(|I(p)| \cdot M_0(p) - |I(p)| \cdot M(p)) > 0$. We do have the problem to verify this proposition because it is of exponential complexity to enumerate all states, i.e., $M \in R(N, M_0)$. In practice, we can substitute $R(N, M_0)$ by $M = M_0 + [N] \cdot \overrightarrow{\sigma}$, thus leading to a mathematical programming (MP) problem

$$\min \sum_{p\in\|I\|^+\cap\{P\backslash S\}} (|I(p)| \cdot M_0(p) - |I(p)| \cdot M(p)) \quad (1)$$

$$\text{s.t. } M = M_0 + [N] \cdot \overrightarrow{\sigma}. \quad (2)$$

If the above MP problem results in a positive objective, the third condition is true; otherwise, we need to resort to other assess criteria. This implies that the verification on the basis of the state equation is a sufficient condition, which nevertheless does not necessarily decrease the contribution of this paper.

C. Case Study

Taking PN in Fig. 1 as an example, we discuss various applications of the above theoretical results.

Case 1: S_1 and S_2 are the elementary siphons, and S_3 is their strongly dependent one. Therefore, we have $\gamma_{S_3} = \gamma_{S_1} + \gamma_{S_2}$. According to Theorem 3, we predict that $l_3 = l_1 + l_2$. This can be easily verified because $l_3 = [0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0]^T = l_1 + l_2 = [0\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0]^T + [0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0]^T$.

According to Proposition 3, this case conforms to the P-invariant control principle. Therefore, S_3 can be implicitly controlled when S_1 and S_2 are P-invariant controlled and their monitors' initial tokens are properly adjusted such that $M_0(S_3) > (M_0(S_1) - \xi_{S_1}) + (M_0(S_2) - \xi_{S_2}) + \Delta_{S_3}$. Hereby, we have $M_0(S_3) = 6$, $M_0(S_1) = 4$, $M_0(S_2) = 4$, $\xi_{S_1} = 2$, $\xi_{S_2} = 2$, and $\Delta_{S_3} = 1$ implying $6 \ge (4-2)+(4-2)+1$. This indicates no need to further increase ξ_{S_1} and ξ_{S_2}. In semantics of inequality analysis, we have $l_3 \le l_1 + l_2$ and $b_3 = 4 \ge 4 = b_1 + b_2$. This exactly follows Theorem 2 such that $l_3^T \cdot M \le b_3$ is dependent on $l_1^T \cdot M \le b_1$ and $l_2^T \cdot M \le b_2$.

Case 2 (Resp., Case 3): S_2 and S_3 (resp., S_2 and S_3) are the elementary siphons and S_1 (resp., S_2) is their weakly dependent one. Therefore, we have $\gamma_{S_1} = \gamma_{S_3} - \gamma_{S_2}$ (resp., $\gamma_{S_2} = \gamma_{S_3} - \gamma_{S_1}$). According to Theorem 4, we predict that $l_1 = l_3 - l_2$ (resp., $l_2 = l_3 - l_1$). This can be easily verified because $l_1 = [0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0]^T = l_3 - l_2 = [0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0]^T - [0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0]^T$ (resp., $l_2 = [0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0]^T = l_3 - l_1 = [0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0]^T - [0\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0]^T$).

According to Proposition 4, this case does not conform to the P-invariant control principle. Therefore, the elementary siphons approach cannot well explain the implicit control of S_1 (resp., S_2) dependent on S_2 and S_3 (resp., S_1 and S_3). Nevertheless, our inequality analysis technique still works. Considering $\gamma_{S_1} = \gamma_{S_3} - \gamma_{S_2}$ (resp., $\gamma_{S_2} = \gamma_{S_3} - \gamma_{S_1}$), we have $l_1 = l_3 - l_2$ (resp., $l_2 = l_3 - l_1$) implying $l_1 \leq l_3$ (resp., $l_2 \leq l_3$). Since $b_3 = 4 > 2 = b_1$ (resp., $b_3 = 4 > 2 = b_2$), we need to decrease b_3 to 2 so that $b_3 = 2 \leq 2 = b_1$ (resp., $b_3 = 2 \leq 2 = b_2$). By such an operation, $l_1^T \cdot M \leq b_1$ (resp., $l_2^T \cdot M \leq b_2$) is implicitly implemented by $l_3^T \cdot M \leq b_3$. Although it does not conform to the P-invariant control principle, this case does conform to our newly created generalized P-invariant control principle in Proposition 5. This means that a weakly dependent siphon can be implicitly controlled by ignoring those elementary siphons corresponding to the positive members in the control-oriented P-invariant, e.g., I. In nature, this mechanism is equivalent to our inequality analysis; nevertheless, the latter is much simpler to comprehend and execute.

Suppose there is no S_3. We have two GMECs, i.e., $l_1^T \cdot M \leq b_1 \Leftrightarrow M(p_2) + M(p_7) \leq 2$ and $l_2^T \cdot M \leq b_2 \Leftrightarrow M(p_3) + M(p_6) \leq 2$. Although $b_1 = 2 \geq b_2 = 2$ and $b_2 = 2 \geq b_1 = 2$, we have neither $l_1 \leq l_2$ nor $l_2 \leq l_1$ because they have distinct components. In this case, we have two options. First, $l_1^T \cdot M \leq b_1$ and $l_2^T \cdot M \leq b_2$ are independent. There is no dependent inequality to remove. Second, we can add either $M(p_3) + M(p_6)$ to the left-hand side of $l_1^T \cdot M \leq b_1$ or $M(p_2) + M(p_7)$ to the left-hand side of $l_2^T \cdot M \leq b_2$. Both lead to $l'^T \cdot M \leq b' \Leftrightarrow M(p_2) + M(p_3) + M(p_6) + M(p_7) \leq 2$. Because $l_1 \leq l'$ or $l_2 \leq l'$ as well as $b_1 \geq b'$ or $b_2 \geq b'$, we can identify $l_1^T \cdot M \leq b_1$ or $l_2^T \cdot M \leq b_2$ as dependent inequalities. As a result, $l'^T \cdot M \leq b'$ is the independent one.

To attain a live system but avoid cumbersome iterative computation, each monitor's outgoing arcs are attached to the source transition, i.e., $t_0^i \in p_{0i}^\bullet$ [9]. By t_0^i, we mean the source transition that is the first transition in the ith process. This operation aims to avoid the emergence of new siphons partially or completely constituted by monitors. In the paradigm of elementary siphons, such an operation does destroy the implicit control mechanism for both strongly and weakly dependent siphons, which has been proven in Propositions 3–5. This is largely because these siphons' corresponding weighted T-characteristic vectors have been deliberately modified. In a weighted T-characteristic vector, a positive (resp., negative) number actually denotes an ingoing (resp., outgoing) arc and its weight from a transition (resp., monitor) to a monitor (resp., transition). These vectors have been modified during the movement of an arc's end from its present transition to a source one. In the framework of our inequality analysis, there is no such an antinomy because all these structural changes are promptly updated in the inequalities. Therefore, no paradox will occur.

TABLE I
SUPERVISOR FOR PN IN FIG. 1

Method	i	$M_0(p_{c_i})$	$^\bullet p_{c_i}$	$p_{c_i}^\bullet$
Approach in [13]	1	3	$\{t_2, t_6\}$	$\{t_1, t_5\}$
Approach in [30]	1	2	$\{t_3, t_6\}$	$\{t_2, t_5\}$
	2	4	$\{t_4, t_8\}$	$\{t_1, t_6\}$
	3	3	$\{t_2, t_7\}$	$\{t_1, t_5\}$
	4	2	$\{t_2, t_7\}$	$\{t_1, t_6\}$
	5	3	$\{t_3, t_6\}$	$\{t_1, t_5\}$
	6	3	$\{t_3, t_7\}$	$\{t_1, t_5\}$
Approach in [14]	×	×	×	×

For some approaches, they realize the system liveness without considering siphons. In [30], a nearly optimal supervisor can be achieved by iteratively identifying these so-called first-met-bad (FMB) markings. Each time, an FMB marking is found. To avoid it along with its subsequent markings, a GMEC inequality is created to constrain the overburden of these processes. Because it precisely removes these FMB markings, such an approach achieves a supervisor with a nearly maximally permissive behavior. Apparently, there is no concept of siphons during this policy's execution. So, the elementary-siphon-based technique cannot be applied to simplify such supervisors. For our inequality analysis, we reduce the number of inequalities in a purely algebraic way. Therefore, it still can work effectively hereby.

For the PN in Fig. 1, we apply the approach in [30]; there are six inequalities being created in sequence. They are $M(p_{11}) + M(p_{21}) \leq 3$, $M(p_{12}) + M(p_{21}) \leq 2$, $M(p_{11}) + M(p_{21}) + M(p_{22}) \leq 3, M(p_{11}) + M(p_{22}) \leq 2, M(p_{11}) + M(p_{12}) + M(p_{21}) \leq 3$, and $M(p_{11}) + M(p_{12}) + M(p_{21}) + M(p_{22}) \leq 3$. There are six GMECs, i.e., $l_1^T \cdot M \leq b_1 \Leftrightarrow M(p_{11}) + M(p_{21}) \leq 3$, $l_2^T \cdot M \leq b_2 \Leftrightarrow M(p_{12}) + M(p_{21}) \leq 2$, $l_3^T \cdot M \leq b_3 \Leftrightarrow M(p_{11}) + M(p_{21}) + M(p_{22}) \leq 3$, $l_4^T \cdot M \leq b_4 \Leftrightarrow M(p_{11}) + M(p_{22}) \leq 2$, $l_5^T \cdot M \leq b_5 \Leftrightarrow M(p_{11}) + M(p_{12}) + M(p_{21}) \leq 3$, and $l_6^T \cdot M \leq b_6 \Leftrightarrow M(p_{11}) + M(p_{12}) + M(p_{21}) + M(p_{22}) \leq 3$. After we decrease b_6 from $b_6 = 3$ to $b_6' = 2$, we have $l_1 \leq l_6$, $l_2 \leq l_6$, $l_3 \leq l_6$, $l_4 \leq l_6$, and $l_5 \leq l_6$ as well as $b_1 \geq b_6'$, $b_2 \geq b_6'$, $b_3 \geq b_6'$, $b_4 \geq b_6'$, and $b_5 \geq b_6'$. According to Theorem 2, we apparently can identify $l_6^T \cdot M \leq b_6' \Leftrightarrow M(p_{11}) + M(p_{12}) + M(p_{21}) + M(p_{22}) \leq 2$ as the unique independent inequality. Apparently, we can apply Theorem 2 to retrieve an independent inequality, i.e., $M(p_{11}) + M(p_{12}) + M(p_{21}) + M(p_{22}) \leq 2$, which can dramatically reduce the number of monitors from six to one. Since there is no concept of siphons, the elementary-siphon-based techniques cannot be applied, thus leading to no solution. In Table I, there are two supervisors derived based on the approaches in [30] and our inequality approach. An interesting issue is that, for this PN, it is with 87 states; among them, 21 are bad and 66 are good. With the supervisor in [30], we use six monitors to achieve 52 (52/66 ≈ 79%) good ones. By our simplified supervisor, we use one monitor to achieve 36 (36/66 ≈ 55%) good ones. How to further preserve more good states remains an open problem to explore.

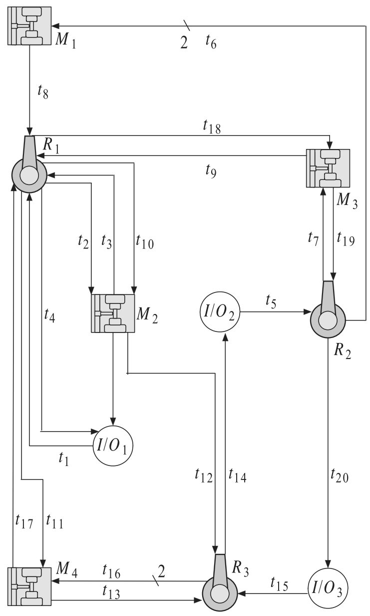

Fig. 2. Block diagram of an FMS.

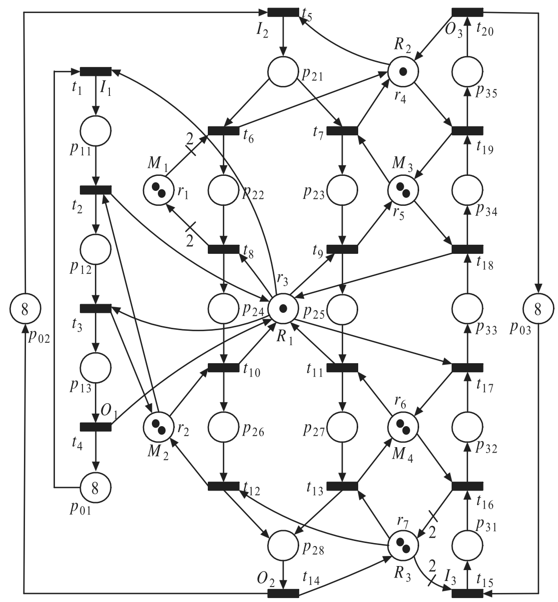

Fig. 3. PN model of the FMS in Fig. 2.

V. ILLUSTRATIVE EXAMPLE

In order to further demonstrate our theoretical achievements, we apply our above results in an even larger FMS whose block diagram is shown in Fig. 2. This layout explains that there are three product types, i.e., $\mathcal{J}_1, \ldots, \mathcal{J}_3$, to be and/or being manufactured. Apart from loading/unloading buffers, this system mainly involves three robots, i.e., $\mathbf{R}_1, \ldots, \mathbf{R}_3$, and four machines, i.e., $\mathbf{M}_1, \ldots, \mathbf{M}_4$. Except $\mathbf{R}_3$ that has two slots, the other robots have only one, which means that one part can be held in the meanwhile. For any $\mathbf{M}_i$, $i \in \mathbb{N}_4 \setminus \{1\}$, two parts can be processed simultaneously, when necessary. To be loaded and unloaded, this FMS interfaces with the warehouse through three pairs of loading and unloading buffers, i.e., $\mathbf{I}_1/\mathbf{O}_1, \ldots, \mathbf{I}_3/\mathbf{O}_3$. In this scenario, robots serve as conveyors so as to transfer parts among various manufacturing regions, which, for $\mathbf{R}_1$ are $\mathbf{I}_1$, $\mathbf{O}_1$, and $\mathbf{M}_1$–$\mathbf{M}_4$; for $\mathbf{R}_2$ are $\mathbf{I}_2$, $\mathbf{O}_3$, $\mathbf{M}_1$, and $\mathbf{M}_3$; and for $\mathbf{R}_3$ are $\mathbf{I}_3$, $\mathbf{O}_2$, $\mathbf{M}_2$, and $\mathbf{M}_4$. As a whole, these resources constitute a concurrent system to process $\mathcal{J}_1, \ldots, \mathcal{J}_3$ in parallel.

Apparently, interactions among these resources can be varying either offline or online, making the FMS flexible so as to be responsive to fluctuating market demand and changing custom requirement. Despite its potential reconfigurability, this FMS is architecturally unchanged during analysis, implying determined progress routes and resource acquisition. For $\mathcal{J}_1$, its raw and finished products are loaded from $\mathbf{I}_1$ and unloaded to $\mathbf{O}_1$ by $\mathbf{R}_1$, respectively. In between, a fabrication is on $\mathbf{M}_2$. For $\mathcal{J}_2$, its raw and finished products are loaded from $\mathbf{I}_2$ and unloaded to $\mathbf{O}_2$ by $\mathbf{R}_1$ and $\mathbf{R}_3$, respectively. In between, there are two optional routes for $\mathcal{J}_2$'s fabrication. One is $\mathbf{M}_1 \rightarrow \mathbf{R}_1 \rightarrow \mathbf{M}_2$, which means that $\mathcal{J}_2$ is treated by $\mathbf{M}_1$ and $\mathbf{M}_2$ in sequence. Another is $\mathbf{M}_3 \rightarrow \mathbf{R}_1 \rightarrow \mathbf{M}_4$, which means that $\mathcal{J}_2$ is treated by $\mathbf{M}_3$ and $\mathbf{M}_4$ in sequence. Between $\mathbf{M}_1$ and $\mathbf{M}_2$ or $\mathbf{M}_3$ and $\mathbf{M}_4$, $\mathbf{R}_1$ performs the conveying operation from the former, i.e., $\mathbf{M}_1$ or $\mathbf{M}_2$, to the latter, i.e., $\mathbf{M}_3$ or $\mathbf{M}_4$. At the end of either route, the final product of $\mathcal{J}_2$ is obtained. For $\mathcal{J}_3$, its raw and finished products are loaded from $\mathbf{I}_3$ and unloaded to $\mathbf{O}_3$ by $\mathbf{R}_3$ and $\mathbf{R}_2$, respectively. In between, $\mathcal{J}_3$ is treated by $\mathbf{M}_4$ and $\mathbf{M}_3$ in sequence where $\mathbf{R}_1$ performs the conveying operation from the former to the latter. A notable issue is that two copies of resources are required when $\mathbf{M}_1$ copes with $\mathcal{J}_2$ and when $\mathbf{R}_3$ conveys $\mathcal{J}_3$ from $\mathbf{I}_3$ to $\mathbf{M}_4$. For others, this quantity maintains one.

Fig. 2 shows this FMS' layout where the directed arcs denote resource allocation events with regard to t_i, $i \in \mathbb{N}_{20}$. Across each arc, there is a number representing the requested or released resource quantity. By default, its value is one.

Since it allows multiple resource acquisitions and flexible routes, this FMS's PN model is an S^4R, as shown in Fig. 3, where $P_0 = \{p_{01}, p_{02}, p_{03}\}$, $P_{A_1} = \{p_{11}, \ldots, p_{13}\}$, $P_{A_2} = \{p_{21}, \ldots, p_{28}\}$, $P_{A_3} = \{p_{31}, \ldots, p_{35}\}$, $P_R = \{r_1, \ldots, r_7\}$, $t_0^1 = t_1$, $t_0^2 = t_5$, and $t_0^3 = t_{15}$. Note, by t_0^i, we mean the source place that is the first transition in the ith process. Formally, we have $t_0^i \in p_{0i}^{\bullet}$. Moreover, $r_1, \ldots, r_7$ correspond to $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{R}_1$, $\mathbf{R}_2$, $\mathbf{M}_3$, $\mathbf{R}_4$, and $\mathbf{R}_3$, respectively. At the initial state, no part is being processed, implying $M_0(P \setminus \{P_0 \cup P_R\}) = 0$. Without loss of generality, we assume that at most eight job instances are allowed for a part type $\mathcal{J}_1, \ldots, \mathcal{J}_3$ at a time, respectively. Thus, $M_0(p_{01}) = M_0(p_{02}) = M_0(p_{03}) = 8$.

This net is deadlock prone since some siphons can be eventually undermarked during system evolution. Our analysis shows that there are 18 siphons, i.e., $S_1 = \{p_{28}, p_{32}, r_6, r_7\}$, $S_2 = \{p_{13}, p_{28}, p_{35}, r_1, \ldots, r_7\}$, $S_3 = \{p_{13}, p_{28}, p_{34}, r_2, r_3, r_5, \ldots, r_7\}$, $S_4 = \{p_{13}, p_{28}, p_{33}, r_2, r_3, r_6, r_7\}$, $S_5 = \{p_{13}, p_{26}, p_{27}, p_{35}, r_1, \ldots, r_6\}$, $S_6 = \{p_{13}, p_{26}, p_{27}, p_{34}, r_2, r_3, r_5, r_6\}$, $S_7 = \{p_{13}, p_{26}, p_{27}, p_{33}, r_2, r_3, r_6\}$, $S_8 = \{p_{13}, p_{25}, p_{26}, p_{35}, r_1, \ldots, r_5\}$, $S_9 = \{p_{13}, p_{25}, p_{26}, p_{34}, r_2, r_3, r_5\}$, $S_{10} = \{p_{13}, p_{25}, p_{26}, p_{33}, r_2, r_3\}$, $S_{11} = \{p_{11}, p_{13}, p_{24}, p_{28}, p_{35}, r_1, \ldots, r_7\}$, $S_{12} = \{p_{11}, p_{13}, p_{24}, p_{27}, p_{35}, r_1, r_3, \ldots, r_6\}$, $S_{13} = \{p_{11}, p_{13}, p_{24}, p_{28}, p_{34}, r_3, r_5, \ldots, r_7\}$,

TABLE II

GENERATED INEQUALITIES CORRESPONDING TO SIPHONS

i	Inequalities ($l_i^T \cdot M \leq b_i$)	Independent
1	$M(p_{21}) + M(p_{23}) + M(p_{25}) + M(p_{27}) + 2{\cdot}M(p_{31}) \leq 2$	No
2	$M(p_{11}){+}M(p_{12}){+}2{\cdot}M(p_{21}){+}2{\cdot}M(p_{22}){+}M(p_{23}){+}\ldots{+}M(p_{27}){+}2{\cdot}M(p_{31}){+}M(p_{32}){+}M(p_{33}){+}M(p_{34}){\leq}10$	Yes
3	$M(p_{11}){+}M(p_{12}){+}M(p_{21}){+}M(p_{22}){+}M(p_{23}){+}M(p_{24}){+}M(p_{25}){+}M(p_{26}){+}M(p_{27}){+}2{\cdot}M(p_{31}){+}M(p_{32}){+}M(p_{33}){\leq}7$	No
4	$M(p_{11}){+}M(p_{12}){+}M(p_{21}){+}M(p_{22}){+}M(p_{23}){+}M(p_{24}){+}M(p_{25}){+}M(p_{26}){+}M(p_{27}){+}2{\cdot}M(p_{31}){+}M(p_{32}){+}M(p_{33}){\leq}10$	No
5	$M(p_{11}){+}M(p_{12}){+}2{\cdot}M(p_{21}){+}2{\cdot}M(p_{22}){+}M(p_{23}){+}M(p_{24}){+}M(p_{25}){+}M(p_{31}){+}M(p_{32}){+}M(p_{33}){+}M(p_{34}){\leq}8$	No
6	$M(p_{11}){+}M(p_{12}){+}M(p_{21}){+}M(p_{22}){+}M(p_{23}){+}M(p_{24}){+}M(p_{25}){+}M(p_{31}){+}M(p_{32}){+}M(p_{33}){\leq}6$	No
7	$M(p_{11}){+}M(p_{12}){+}M(p_{21}){+}M(p_{22}){+}M(p_{23}){+}M(p_{24}){+}M(p_{25}){+}M(p_{31}){+}M(p_{32}){\leq}4$	No
8	$M(p_{11}){+}M(p_{12}){+}2{\cdot}M(p_{21}){+}2{\cdot}M(p_{22}){+}M(p_{23}){+}M(p_{24}){+}M(p_{31}){+}M(p_{32}){+}M(p_{33}){+}M(p_{34}){\leq}6$	No
9	$M(p_{11}){+}M(p_{12}){+}M(p_{21}){+}M(p_{22}){+}M(p_{23}){+}M(p_{24}){+}M(p_{31}){+}M(p_{32}){+}M(p_{33}){\leq}4$	No
10	$M(p_{11}){+}M(p_{12}){+}M(p_{21}){+}2{\cdot}M(p_{22}){+}M(p_{24}){\leq}2$	No
11	$2{\cdot}M(p_{21}){+}2{\cdot}M(p_{22}){+}M(p_{23}){+}M(p_{25}){+}M(p_{27}){+}2{\cdot}M(p_{31}){+}M(p_{32}){+}M(p_{33}){+}M(p_{34}){\leq}8$	No
12	$2{\cdot}M(p_{21}){+}2{\cdot}M(p_{22}){+}M(p_{23}){+}M(p_{25}){+}M(p_{31}){+}M(p_{32}){+}M(p_{33}){+}M(p_{34}){\leq}6$	No
13	$M(p_{21}){+}M(p_{23}){+}M(p_{25}){+}M(p_{27}){+}2{\cdot}M(p_{31}){+}M(p_{32}){+}M(p_{33}){\leq}5$	No
14	$M(p_{21}){+}M(p_{23}){+}M(p_{25}){+}M(p_{31}){+}M(p_{32}){+}M(p_{33}){\leq}4$	No
15	$M(p_{21}){+}M(p_{23}){+}M(p_{25}){+}M(p_{27}){+}2{\cdot}M(p_{31}){+}M(p_{32}){\leq}3$	No
16	$M(p_{21}){+}M(p_{23}){+}M(p_{25}){+}M(p_{31}){+}M(p_{32}){\leq}2$	No
17	$2{\cdot}M(p_{21}){+}2{\cdot}M(p_{22}){+}M(p_{23}){+}M(p_{31}){+}M(p_{32}){+}M(p_{33}){+}M(p_{34}){\leq}4$	No
18	$M(p_{21}){+}M(p_{23}){+}M(p_{31}){+}M(p_{32}){+}M(p_{33}){\leq}2$	No

$S_{14} = \{p_{11}, p_{13}, p_{24}, p_{27}, p_{34}, r_3, r_5, r_6\}$, $S_{15} = \{p_{11}, p_{13}, p_{24}, p_{28}, p_{33}, r_3, r_6, r_7\}$, $S_{16} = \{p_{11}, p_{13}, p_{24}, p_{27}, p_{33}, r_3, r_6\}$, $S_{17} = \{p_{11}, p_{13}, p_{24}, p_{25}, p_{35}, r_1, r_3, \ldots, r_5\}$, and $S_{18} = \{p_{11}, p_{13}, p_{24}, p_{25}, p_{34}, r_3, r_5\}$.

Using Definition 6, we can verify that $S_1, \ldots, S_5$ and S_8 are elementary siphons, while S_6, S_7, and $S_9, \ldots, S_{18}$ are their dependent ones such that $\gamma_{S_6} = \gamma_{S_3} + \gamma_{S_5} - \gamma_{S_2}$, $\gamma_{S_7} = \gamma_{S_4} + \gamma_{S_5} - \gamma_{S_2}$, $\gamma_{S_9} = \gamma_{S_3} + \gamma_{S_8} - \gamma_{S_2}$, $\gamma_{S_{10}} = \gamma_{S_4} + \gamma_{S_8} - \gamma_{S_2}$, $\gamma_{S_{11}} = \gamma_{S_1} + \gamma_{S_2} + \gamma_{S_5} - \gamma_{S_4} - \gamma_{S_8}$, $\gamma_{S_{12}} = \gamma_{S_2} + \gamma_{S_5} - \gamma_{S_4} - \gamma_{S_8}$, $\gamma_{S_{13}} = \gamma_{S_1} + \gamma_{S_3} + \gamma_{S_5} - \gamma_{S_4} - \gamma_{S_8}$, $\gamma_{S_{14}} = \gamma_{S_3} + \gamma_{S_5} - \gamma_{S_4} - \gamma_{S_8}$, $\gamma_{S_{15}} = \gamma_{S_1} + \gamma_{S_5} - \gamma_{S_8}$, $\gamma_{S_{16}} = \gamma_{S_5} - \gamma_{S_8}$, $\gamma_{S_{17}} = \gamma_{S_2} - \gamma_{S_4}$, and $\gamma_{S_{18}} = \gamma_{S_3} - \gamma_{S_4}$.

To verify Theorems 3 and 4, we list $l_{S_1} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 2\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_2} = [0\ 1\ 1\ 0\ 0\ 1\ 2\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 2\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_3} = [0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 2\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_4} = [0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 2\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_5} = [0\ 1\ 1\ 0\ 0\ 1\ 2\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_6} = [0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_7} = [0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_8} = [0\ 1\ 1\ 0\ 0\ 1\ 2\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_9} = [0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_{10}} = [0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_{11}} = [0\ 0\ 0\ 0\ 0\ 1\ 2\ 1\ 0\ 1\ 0\ 1\ 0\ 0\ 2\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_{12}} = [0\ 0\ 0\ 0\ 0\ 1\ 2\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_{13}} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ 0\ 2\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_{14}} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_{15}} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 2\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_{16}} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $l_{S_{17}} = [0\ 0\ 0\ 0\ 0\ 1\ 2\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, and $l_{S_{18}} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$.

Since $\gamma_{S_6} = \gamma_{S_3} + \gamma_{S_5} - \gamma_{S_2}$, we can verify that $l_{S_6} = [0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T = l_{S_3} + l_{S_5} - l_{S_2} = [0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 2\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T + [0\ 1\ 1\ 0\ 0\ 1\ 2\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T - [0\ 1\ 1\ 0\ 0\ 1\ 2\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 2\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$. Analogously, other equations can be validated. $l_{S_7} = l_{S_4} + l_{S_5} - l_{S_2}$, $l_{S_9} = l_{S_3} + l_{S_8} - l_{S_2}$, $l_{S_{10}} = l_{S_4} + l_{S_8} - l_{S_2}$, $l_{S_{11}} = l_{S_1} + l_{S_2} + l_{S_5} - l_{S_4} - l_{S_8}$, $l_{S_{12}} = l_{S_2} + l_{S_5} - l_{S_4} - l_{S_8}$, $l_{S_{13}} = l_{S_1} + l_{S_3} + l_{S_5} - l_{S_4} - l_{S_8}$, $l_{S_{14}} = l_{S_3} + l_{S_5} - l_{S_4} - l_{S_8}$, $l_{S_{15}} = l_{S_1} + l_{S_5} - l_{S_8}$, $l_{S_{16}} = l_{S_5} - l_{S_8}$, $l_{S_{17}} = l_{S_2} - l_{S_4}$, and $l_{S_{18}} = l_{S_3} - l_{S_4}$. By considering $l_{S_i} \geq 0^T$, all equations can be converted to inequalities, i.e., $l_{S_7} \leq l_{S_4} + l_{S_5}$, $l_{S_9} \leq l_{S_3} + l_{S_8}$, $l_{S_{10}} \leq l_{S_4} + l_{S_8}$, $l_{S_{11}} \leq l_{S_1} + l_{S_2} + l_{S_5}$, $l_{S_{12}} \leq l_{S_2} + l_{S_5}$, $l_{S_{13}} \leq l_{S_1} + l_{S_3} + l_{S_5}$, $l_{S_{14}} \leq l_{S_3} + l_{S_5}$, $l_{S_{15}} \leq l_{S_1} + l_{S_5}$, $l_{S_{16}} \leq l_{S_5}$, $l_{S_{17}} \leq l_{S_2}$, and $l_{S_{18}} \leq l_{S_3}$. Obviously, in terms of elementary siphons, their corresponding inequalities implicitly match the prerequisite for the inequality dependence shown in Theorem 2, i.e., $l_k \leq \sum_{i \in \mathbb{N}_n \setminus \{k\}} \alpha_i \cdot l_i$. In the case that we properly reduce the right-hand scalars of these inequalities with regard to elementary siphons, we also fulfil another prerequisite in Theorem 2, i.e., $b_k \geq \sum_{i \in \mathbb{N}_n \setminus \{k\}} \alpha_i \cdot b_i$. This phenomenon discloses that elementary siphon is absolutely a special case of our proposed supervisor simplification techniques using inequality analysis. In other words, our inequality analysis strategy can well explain the rationale behind of elementary siphons, but not vice versa.

To achieve live systems, conventional method like that in [7] requires the control of each siphons, leading to 18 inequalities as shown in Table II for the net in Fig. 3. For the sake of brevity, $p_c^\bullet$ is assumed to be t_0, as done in [9] and in the sequel. From the perspective of structural control, this means 18 monitors, i.e., $p_{c_1}, \ldots, p_{c_{18}}$, with regard to $S_1, \ldots, S_{18}$, respectively. For clarity, they are shown in Table III. Liveness property is achieved with these 18 monitors. Our analysis shows that the controlled system is live with 4949 reachable states.

In terms of elementary siphons, 6 siphons, i.e., $S_1, \ldots, S_5$ & S_8, among these 18 ones, i.e., $S_1, \ldots, S_{18}$, are elementary ones on which the remaining others, i.e., S_6, S_7 & $S_9, \ldots, S_{18}$, weakly depend based on our analysis, simplifying the control object to a supervisor with six monitors, as shown in Table IV, which makes our system live with 109 reachable states.

Using our inequality analysis techniques, the second inequality, $l_{S_2}^T \cdot M \leq b_2$, i.e., $M(p_{11}) + M(p_{12}) + 2 \cdot M(p_{21})$

TABLE III

GENERATED MONITORS FOR THE NET IN FIG. 3 DUE TO [7]

i	$^\bullet p_{c_i}$	$p_{c_i}^\bullet$	$M_0(p_{c_i})$	i	$^\bullet p_{c_i}$	$p_{c_i}^\bullet$	$M_0(p_{c_i})$
1	$\{t_6, t_{13}, 2\cdot t_{16}\}$	$\{t_5, 2\cdot t_{15}\}$	2	10	$\{t_3, t_7, t_{10}\}$	$\{t_1, t_5\}$	2
2	$\{t_3, t_7, t_8, t_{12}, t_{13}, t_{16}, t_{19}\}$	$\{t_1, 2\cdot t_5, 2\cdot t_{15}\}$	10	11	$\{t_7, 2\cdot t_8, t_{13}, t_{16}, t_{19}\}$	$\{2\cdot t_5, 2\cdot t_{15}\}$	8
3	$\{t_3, t_{12}, t_{13}, t_{16}, t_{18}\}$	$\{t_1, t_5, 2\cdot t_{15}\}$	7	12	$\{t_7, 2\cdot t_8, t_{11}, t_{19}\}$	$\{2\cdot t_5, t_{15}\}$	6
4	$\{t_3, t_{12}, t_{13}, t_{16}, t_{17}\}$	$\{t_1, t_5, 2\cdot t_{15}\}$	5	13	$\{t_6, t_{13}, t_{16}, t_{18}\}$	$\{t_5, 2\cdot t_{15}\}$	5
5	$\{t_3, t_7, t_8, t_{10}, t_{11}, t_{19}\}$	$\{t_1, 2\cdot t_5, t_{15}\}$	8	14	$\{t_6, t_{11}, t_{18}\}$	$\{t_5, t_{15}\}$	4
6	$\{t_3, t_{10}, t_{11}, t_{18}\}$	$\{t_1, t_5, t_{15}\}$	6	15	$\{t_6, t_{13}, t_{16}, t_{17}\}$	$\{t_5, 2\cdot t_{15}\}$	3
7	$\{t_3, t_{10}, t_{11}, t_{17}\}$	$\{t_1, t_5, t_{15}\}$	4	16	$\{t_6, t_{11}, t_{17}\}$	$\{t_5, t_{15}\}$	2
8	$\{t_3, t_7, t_8, t_9, t_{10}, t_{19}\}$	$\{t_1, 2\cdot t_5, t_{15}\}$	6	17	$\{t_7, 2\cdot t_8, t_9, t_{19}\}$	$\{2\cdot t_5, t_{15}\}$	4
9	$\{t_3, t_9, t_{10}, t_{18}\}$	$\{t_1, t_5, t_{15}\}$	4	18	$\{t_6, t_9, t_{18}\}$	$\{t_5, t_{15}\}$	2

$+ 2\cdot M(p_{22}) + M(p_{23}) + M(p_{24}) + M(p_{25}) + M(p_{26}) + M(p_{27}) + 2\cdot M(p_{31}) + M(p_{32}) + M(p_{33}) + M(p_{34}) \le 10$, is independent after we decrease b_2 from 10 to 2. Actually, Theorem 2 provides a quite straightforward way to justify so, i.e., $\forall i \in \mathbb{N}_{18} \setminus \{2\}$, we have $l_i^T \le l_2$ and $b_i \ge b_2 = 2$. From SCT perspective, this inequality results in a supervisor as shown in Fig. 4, leading to a live system with 109 reachable states.

As observed, elementary siphons provide a quite conservative system behavior with a moderate-sized supervisor. Our inequality analysis technique can easily identify and implement the simplest supervisor with the same behavior. This is quite beyond the capability of elementary siphons because they cannot explain the theoretical rationale behind of this simplification procedures. Consider our resultant supervisor in Fig. 4. Despite the equivalence in permissiveness with regard to the strategies in [7] and [14], ours can significantly reduce the monitor quantity to one, which is $6\% \approx 1/18$ and $17\% \approx 1/6$ compared with the ones obtained from [7] and [14], respectively. More importantly, our proposed inequality analysis techniques can well explore all theoretical rationales behind of these simplification operations, which are far beyond elementary siphons' explanation capability.

The calculation of elementary siphons strictly follows Definitions 5 and 6. For detailed procedures, the readers are referred to [20], where the elementary-siphon-based technique is initially proposed, and [14], where such a technique is extended for its adaption in the general PNs, e.g., S^4R.

Reference [20] claims that the control of elementary siphons can lead to more permissiveness. Nevertheless, this is only a subjective assertion without any justification. Their major argument is that the elementary-siphon-based technique can decrease the number of constraints, especially compared with [9]. However, a smaller number of constraints do not necessarily imply a more permissive behavior. It also highly depends on how tightly each constraint will exert on the original system. Unfortunately, we observe many such evidences.

1) When applying the elementary-siphon-based technique, it seems that only elementary siphons are controlled. However, the fact is not as simple as so. Each elementary siphon's dependent siphons must be controlled implicitly. To achieve this, one has to decrease the so-called control depth whose involvement will significantly decrease the initial markings of control places and downgrade the being-controlled PN's permissiveness. In a usual tongue, each elementary siphon is never optimally controlled by considering itself independently. A quite tight constraint must be involved to control all other siphons sharing partial or total complementary set with these particular elementary siphons. This is a secret behind of elementary-siphon-based technique and cannot be overlooked.
2) To implement the control of elementary siphons along with their dependent ones, [20] has to attach the outgoing arcs of each control place to their corresponding source transitions, i.e., the transitions exactly after the idle places ($t_0^i \in p_{0i}^\bullet$). Inherited from [9], such a scheme imposes a quite tight constraint to every elementary siphon. This is because it not only constrains the token number in each elementary

TABLE IV

GENERATED MONITORS FOR THE NET IN FIG. 3 DUE TO ELEMENTARY SIPHONS

i	$^\bullet p_{c_i}$	$p_{c_i}^\bullet$	$M_0(p_{c_i})$
1	$\{t_6, t_{13}, 2\cdot t_{16}\}$	$\{t_5, 2\cdot t_{15}\}$	2
2	$\{t_3, t_7, t_8, t_{12}, t_{13}, t_{16}, t_{19}\}$	$\{t_1, 2\cdot t_5, 2\cdot t_{15}\}$	2
3	$\{t_3, t_{12}, t_{13}, t_{16}, t_{18}\}$	$\{t_1, t_5, 2\cdot t_{15}\}$	2
4	$\{t_3, t_{12}, t_{13}, t_{16}, t_{17}\}$	$\{t_1, t_5, 2\cdot t_{15}\}$	2
5	$\{t_3, t_7, t_8, t_{10}, t_{11}, t_{19}\}$	$\{t_1, 2\cdot t_5, t_{15}\}$	2
6	$\{t_3, t_7, t_8, t_9, t_{10}, t_{19}\}$	$\{t_1, 2\cdot t_5, t_{15}\}$	2

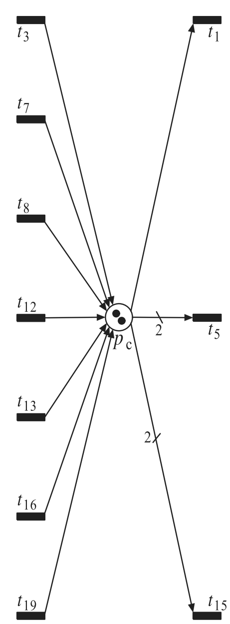

Fig. 4. Supervisor.

TABLE V
GENERATED MONITORS FOR THE NET IN FIG. 3 DUE TO [30]

i	$^\bullet p_{c_i}$	$p_{c_i}^\bullet$	$M_0(p_{c_i})$	i	$^\bullet p_{c_i}$	$p_{c_i}^\bullet$	$M_0(p_{c_i})$
1	$\{t_3\}$	$\{t_1\}$	2	12	$\{t_3, t_9, t_{12}, t_{17}\}$	$\{t_1, t_7, t_{10}, t_{15}\}$	6
2	$\{t_3, t_{10}\}$	$\{t_2, t_8\}$	2	13	$\{t_3, t_{12}, t_{13}, t_{17}\}$	$\{t_1, t_{10}, t_{11}, t_{15}\}$	5
3	$\{t_9, t_{18}\}$	$\{t_7, t_{17}\}$	2	14	$\{t_3, t_{12}, t_{17}\}$	$\{t_2, t_8, t_{15}\}$	5
4	$\{t_{11}, t_{17}\}$	$\{t_9, t_{16}\}$	2	15	$\{t_3, t_9, t_{12}, t_{17}\}$	$\{t_2, t_7, t_8, t_{15}\}$	6
5	$\{t_9, t_{17}\}$	$\{t_7, t_{16}\}$	3	16	$\{t_3, t_{12}, t_{13}, t_{17}\}$	$\{t_2, t_8, t_{11}, t_{15}\}$	5
6	$\{t_{13}, t_{16}\}$	$\{t_{11}, t_{15}\}$	2	17	$\{t_8, t_9, t_{12}, t_{18}\}$	$\{t_5, t_{10}, t_{15}\}$	8
7	$\{t_{11}, t_{17}\}$	$\{t_7, t_{15}\}$	4	18	$\{t_9, t_{12}, t_{17}, t_{19}\}$	$\{t_5, t_{15}, t_{18}\}$	8
8	$\{t_{13}, t_{17}\}$	$\{t_9, t_{15}\}$	3	19	$\{t_8, t_9, t_{12}, t_{13}, t_{18}\}$	$\{t_5, t_{10}, t_{11}, t_{15}\}$	8
9	$\{t_9, t_{13}, t_{17}\}$	$\{t_7, t_{11}, t_{15}\}$	4	20	$\{t_9, t_{12}, t_{13}, t_{17}, t_{19}\}$	$\{t_5, t_{11}, t_{15}, t_{18}\}$	8
10	$\{t_3, t_{12}, t_{17}\}$	$\{t_1, t_{10}, t_{15}\}$	5	21	$\{t_8, t_{11}, t_{12}, t_{17}, t_{19}\}$	$\{t_5, t_{10}, t_{15}, t_{18}\}$	8
11	$\{t_8, t_9, t_{19}\}$	$\{t_5, t_{17}\}$	4	–	–	–	–

TABLE VI
GENERATED INEQUALITIES DUE TO [30]

i	Inequalities ($l_i^T \cdot M \leq b_i$)	Independent
1	$M(p_{11}) + M(p_{12}) \leq 2$	Yes
2	$M(p_{12}) + M(p_{24}) \leq 2$	Yes
3	$M(p_{23}) + M(p_{33}) \leq 2$	Yes
4	$M(p_{25}) + M(p_{32}) \leq 2$	Yes
5	$M(p_{23}) + M(p_{32}) \leq 3$	Yes
6	$M(p_{27}) + M(p_{31}) \leq 2$	Yes
7	$M(p_{23}) + M(p_{25}) + M(p_{31}) + M(p_{32}) \leq 4$	Yes
8	$M(p_{25}) + M(p_{27}) + M(p_{31}) + M(p_{32}) \leq 3$	Yes
9	$M(p_{23}) + M(p_{27}) + M(p_{31}) + M(p_{32}) \leq 4$	Yes
10	$M(p_{11}) + M(p_{12}) + M(p_{26}) + M(p_{31}) + M(p_{32}) \leq 5$	No
11	$M(p_{21}) + M(p_{22}) + M(p_{23}) + M(p_{33}) + M(p_{34}) \leq 4$	Yes
12	$M(p_{11}) + M(p_{12}) + M(p_{23}) + M(p_{26}) + M(p_{31}) + M(p_{32}) \leq 6$	Yes
13	$M(p_{11}) + M(p_{12}) + M(p_{26}) + M(p_{27}) + M(p_{31}) + M(p_{32}) \leq 5$	Yes
14	$M(p_{12}) + M(p_{24}) + M(p_{26}) + M(p_{31}) + M(p_{32}) \leq 5$	No
15	$M(p_{12}) + M(p_{23}) + M(p_{25}) + M(p_{26}) + M(p_{31}) + M(p_{32}) \leq 6$	Yes
16	$M(p_{12}) + M(p_{24}) + M(p_{26}) + M(p_{27}) + M(p_{31}) + M(p_{32}) \leq 5$	Yes
17	$M(p_{21}) + M(p_{22}) + M(p_{23}) + M(p_{26}) + M(p_{31}) + M(p_{32}) + M(p_{33}) \leq 8$	No
18	$M(p_{21}) + M(p_{22}) + M(p_{23}) + M(p_{24}) + M(p_{26}) + M(p_{31}) + M(p_{32}) + M(p_{34}) \leq 8$	No
19	$M(p_{21}) + M(p_{22}) + M(p_{23}) + M(p_{26}) + M(p_{27}) + M(p_{31}) + M(p_{32}) + M(p_{33}) \leq 8$	Yes
20	$M(p_{21}) + M(p_{22}) + M(p_{23}) + M(p_{24}) + M(p_{26}) + M(p_{27}) + M(p_{31}) + M(p_{32}) + M(p_{34}) \leq 8$	Yes
21	$M(p_{21}) + M(p_{22}) + M(p_{23}) + M(p_{25}) + M(p_{26}) + M(p_{31}) + M(p_{32}) + M(p_{34}) \leq 8$	Yes

siphon's complementary set but also constrains the token number in other upstream adjacent places between the complementary set and the idle place.

3) In the paradigm of PNs, it is claimed that the number of elementary siphons is bounded by the lower one of the numbers of places and transitions, i.e., $|\mathrm{ES}| \leq \min\{|P|, |T|\}$. However, this claim is true in only one iteration. Given a plant PN model, we can obtain all its siphons, among which some are elementary siphons, as well as others are dependent ones. By following Theorem 1, the control of the former can ensure the same property of the latter. Unfortunately, in the case that we do not attach the outgoing arcs to the source transitions, these control places themselves can lead to new undermarked siphons and, thus, another PN model to be further controlled. This results in an iterative method. At each iteration step, one has to control $|\mathrm{ES}| \leq \min\{|P|, |T|\}$ number of elementary siphons. Even for a PN with moderate size, such an iteration almost can never terminate although one can claim so. This is because the number of states is finite but exponential. In this scenario, elementary-siphon-based technique will introduce more rather than less constraints compared with [9].

To further explore our proposed method's performance, we apply the FMB-oriented techniques in [30] to the same model. After 21 iterations, a supervisor along with its corresponding GMECs is generated as shown in Tables V and VI, respectively. Through the latter, we can easily figure out that inequalities 10, 14, 17, and 18 are dependent on 13, 16, 19, and 20, respectively. Their counterparts in Table V, i.e., monitors 10, 14, 17, and 18, can be deleted. As a result, we attain a supervisor with 17 monitors that provide 18 985 good states. Further analysis shows that the PN in Fig. 3 contains 23 216 states in total among which the bad, dead, and good states' counts are 4213, 120, and 19 003, respectively. Therefore, the inequality analysis approach reduces the monitor quantity by (21–17)/21 $\approx$ 19%, while preserves good states by 18 985/19 003 $\approx$ 99.9%. No simplification result can be achieved by elementary siphon strategy at this moment since there is even no involvement of siphon's concept.

VI. CONCLUSION

This paper is concerned about a comparative study upon some typical supervisor simplification techniques. Their applicability is shown through a general class of FMSs. To well explain such intricate phenomena, some new theoretical results are developed. Our investigation shows that an inequality analysis-based simplification technique can be applied in most scenarios. More importantly, it elegantly describes all supervisor simplification issues from a purely algebraic way. In our future work, more complex FMSs will be investigated. Further investigation is expected to extend our approach from the state-based control domain to the event-based one, where uncontrollable and/or unobservable events are involved so as to make our approach more practical. Maximal permissiveness is another index that should be considered during simplification.

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